Biology 5910: Take home final

Do four out of five problems (25 points each). If you are unhappy with your current grade, do them all and I’ll count the worst problem for extra credit. Open books, notes and basic calculators, but no graphics calculators, computer algebra systems, or other human beings (except Emerson and me). Take no more than 2.5 hours.

1. Ants often associate with aphids. But do they really like each other? Suppose there is an ant colony near a tree with probability 0.9 if there are aphids on that tree, with probability 0.5 if there are no aphids, and that a fraction 0.4 of trees have aphids.

   a. Find the probability that a tree has both ants and aphids.
   b. Find the probability that a randomly chosen tree has ants.
   c. Find the probability of aphids conditional on there being ants.
   d. How could you tell whether ants like to be near aphids, aphids like to be near ants, or both?
2. Suppose the following numbers of ants are observed arriving on a set of trees in 20 minutes.

<table>
<thead>
<tr>
<th>Tree</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>No aphids</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Aphids</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

a. What distribution would you assume these numbers follow, and what are some of the key assumptions?

b. Write the likelihood function for the rate at which ants arrive on trees with no aphids and find the maximum likelihood estimate. What are the units of your estimated parameter?

c. How would you use maximum likelihood to test whether trees with aphids receive more visits than those without?

d. What is another way you could test whether trees with aphids receive more visits than those without?
3. Aphids attract ants by feeding them nutritious secretions (NS). The ants then protect the aphids from predators and parasitoids. However, the ants can also just eat the aphids, and may choose to do so at rate E if they don’t get enough NS. We can think of this as a game where the aphids choose a strategy NS, and the ants a strategy E.

\[
\text{Aphid payoff} = NS(1 - NS - E)
\]
\[
\text{Ant payoff} = \left(\frac{NS}{2} + E\right)(1 - NS - E).
\]

a. What is the best choice of NS for the aphids for a given E by the ants?
b. What is the best choice of E for the ants for a given NS by the aphids?
c. Is there a best reply to a best reply?
d. Is there a strategy which would be better for both ants and aphids?
4. Mathematical modelers have neglected the ant-aphid interaction largely because both begin with the letter A. Let $A_t$ denote the number of aphids on a tree in week $t$, and let $N$ be the number of ants which we assume does not change over the course of a season. Suppose that

$$A_{t+1} = \frac{0.4A_t}{1 + 0.1N} + (1 - \frac{0.5}{1 + N})A_t$$

a. The first term describes aphid reproduction. What effect do ants have? What happens as $N \to \infty$?

b. The second term describes aphid survival. What effect do ants have? What happens as $N \to \infty$?

c. What happens when $N = 0$? Find the equilibrium aphid population and its stability.

d. Find a value of $N$ where something different happens. Find the equilibrium aphid population and its stability.
5. The number of ants visiting a tree does change during the course of a season. Let $A$ represent the number of aphids (in thousands) and $N$ the number of ants (in hundreds) on a given tree, and suppose the numbers are governed by the equations

\[
\frac{dA}{dt} = A(1 - A) - 2A(1 - N)
\]

\[
\frac{dN}{dt} = 0.5A(1 - N).
\]

a. Explain each term in these equations.

b. Suppose the initial conditions are $A(0) = 0.5$ and $N(0) = 0$. Will either of the populations start to grow?

c. Find the equilibrium of the system. What would happen to these populations by the end of the season?