Merton’s Jump Diffusion Model

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Outline

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Background

Terms

- **Option**: (Call/Put) is a derivative written on a security that gives the owner the right to buy or sell the security at a predetermined price on a specified date in the future.
- **Premium**: price of the option.
- **Strike**: the price at which the owner of an option has the right to buy or sell the option.
- **Maturity**: the date on which the option may be exercised.
- **Volatility**: standard deviation of the change in value of an asset over time.
  - In general, the more volatile the asset, the more a derivative contract is worth.
Example: Call Option

- Suppose the strike = $100

Payoff

- If stock price reaches $110 at expiration, then the buyer makes a profit of $10
- If the stock is below $100 at expiration, then it has no value
Build Optimal Portfolio

- Depends on goal: risky v. conservative
- Options
  - By itself it is VERY RISKY
  - When coupled with stock appropriately the portfolio can become LESS RISKY
    - This is called ‘Hedging’
Want to know how a financial product will react in the market

- If stock goes up option moves accordingly.
- Design option to react how you want it to (i.e. calls and put)

This is important because understanding these reactions will help people optimize their portfolios

- i.e. make more money by knowing how to allocate their assets within their investments
Background

- **Black-Scholes Formula**
  - To calculate the fair-market value for any option, the formula uses multiple variables.
  - \( S \) = current stock price,
  - \( K \) = strike price,
  - \( r \) = risk-free interest rate,
  - \( T \) = time to expiration,
  - \( \sigma \) = volatility of the stock.
  - \( t \) = current time
Black-Scholes Formula

\[ C(s, t, \sigma, k) = SN(d_1) - Ke^{-r(T-t)}N(d_2) \]

- where \( N(x) \) is the cumulative normal distribution function
- And \( d_1 \) and \( d_2 \) are defined as:

\[ d_1 = \frac{\log S/K + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \]

\[ d_2 = d_1 - \sigma\sqrt{T-t} \]
Assumptions of Black-Scholes Model

- The stock follows geometric Brownian motion
- No dividends are paid out on the underlying stock during the option life.
- The option can only be exercised at expiration time (European style)
- Efficient markets (No arbitrage)
- No transaction cost
- Risk free interest rates do not change over the life of the option (and are known)
The Problem

- **Black Scholes Model**
  - Requires different volatilities for different strikes and maturities to price the option
- **Ideally want a 1 – 1 correspondence**
  - One volatility assigned to each stock
- **BS assumes a log normal distribution**
  - By adding jumps we correct this flaw
Solution : Merton’s Jump Diffusion Model

- This model attempts to solve the problems associated with a log normal distribution
Research

- Merton’s Jump Diffusion Model
  - Show how Black-Scholes fails
  - Show how Jump Diffusion works
    - Derivation
    - Research using NDX-100 and IBM
Failure of lognormal distribution assumption
Introduction to the Jump Diffusion Model

- Allows for larger moves in asset prices caused by sudden events.
- The jump component represents non-systematic risk, a type of risk that affects a particular company or industry.
Jump Diffusion Model Derivation

- Merton includes a discontinuity of underlying stock returns called a jump.
- The following formula describes a relative change of stock price with the jump factor: $q$

$$\frac{S_{i+1} - S_i}{S_i} = \mu \Delta t + \sigma \Delta \omega_i + \Delta q$$

where

$$\Delta q = \begin{cases} 0 & \text{without jumps} \\ y - 1 & \text{with jumps} \end{cases}$$
Jump Diffusion Model Derivation

- The following is the modified Black-Scholes equation in the jump diffusion model.

\[ \frac{1}{2} \sigma^2 S^2 F_{ss} + (r - \lambda \kappa)SF_s - F_{\tau} - rF + \lambda E[F(sY, \tau) - F(s, \tau)] = 0 \]

- The solution can be represented by a series consisting of terms

\[ F_n = \frac{e^{-\lambda \tau} (\lambda \tau)^n}{n!} E[W(x, \tau, K, \sigma^2)], n = 0, 1, 2, \ldots \]
Volatility Smile

- Implied volatility is the volatility used in Black–Scholes to match the market price.
- The Black-Scholes formula uses different implied volatilities for different strikes and maturities.
- At-the-money options tend to have lower implied volatilities.
Volatility Smile

Volatility increases as the option becomes increasingly in-the-money or out-of-the-money.

- In-the-Money Puts
- Out-of-the-Money Calls
- In-the-Money Calls
- Out-of-the-Money Puts

At-the-Money Calls/Puts

Strike Price ($)

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Volatility smile for an option on the NDX-100

![Volatility Smile](image)

- **Option for the NDX stock**

- **Implied volatility (jump)**
- **Implied volatility (market)**

The University of Utah
Volatility smile for an option on IBM stock

Volatility Smile
(Option for the IBM stock expired on Dec 6th 2006)

- implied vol (jump)
- implied vol (market)
Option Pricing Errors of Jump Diffusion Model and Black-Scholes Model (IBM)

- Price Differences:
  - Jump model: \( \sigma = 0.12 \)
  - BS model: \( \sigma = 0.12 \)

- Strike Prices:
  - Range from 80 to 105
Summary

- Merton included the impact of a sudden large stock fluctuations
- Model works better on individual stocks relative to indices
- Non-systematic jump risk assumption is important in this model.
Future Direction

- Estimate the jump risk on particular stocks and indices
- Analyze the volatility smile to determine systematic or nonsystematic risks