

FINAL REPORT FOR REU
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Osteoporosis is a bone disease in which the amount of bone is decreased and the structural integrity of trabecular bone is impaired. Cortical bone becomes more porous and thinner. This makes the bone weaker and more likely to fracture. Cortical bone is the stiff outer shell of the bone. Trabecular bone is the spongy inner layer. Mathematically, we call bone a heterogeneous random material. A heterogeneous material is just a material that's made of more than one thing. With the bones we're looking at a material of bone and void. We call this a two-phase material for the obvious reason that it has two phases. And we use the term random under the assumption that any given slide is just a realization of a random variable. We assume the structure is independent of time, so that we can look at is a compilation of volume fractions. Volume fractions are disjoint, and the sum of all volume fractions equals the volume of the material, 1. The function that represents the image is just an indicator function. Because I'm most interested in the void space, I let x take of the value of 1 when it lies in the void space and a value of 0 otherwise. Now we're ready to start discussing the mathematical (statistical) characterization of our material. This is done using n -point correlation functions. An n -point correlation function tell us the probability that n vertices of a shape are in a given volume. So, a one-point correlation function is the probability that any one point is in any given volume. We drop a point on the image and get the value. We do this many, many times to get an average. A two-point correlation function is the probability that the endpoints of a line segment lie in any given volumes. So, we drop a line on the image and calculate where the endpoints land. A three-point correlation function is the probability that the vertices of a triangle lie in any given volumes. We drop a triangle on the image and look where the vertices land. This continues in a similar fashion with rectangles, then pentagons, etc. I, however, will focus only on one and two point correlation functions. Now, since $I(x)$ is either the value one or zero, we know that the probability of any point being in either of our phases with also be either the value one or zero. Thus the probabilistic description of $I(x)$ is just $P\{I(x)=1\}$. It follows that $P\{I(x)=0\}=1- P\{I(x)=1\}$. So X can be specified

by the CDF, $F(x)=P\{X<x\}$. We express a one point correlation function, $S(x)$, as the ensemble average over all realizations. Since the expectation is just the same as $P\{I(x)=1\}$, $S(x)=\langle I(x) \rangle = P\{I(x)=1\}$. This one-point correlation function is also sometimes referred to as a one-point probability function, since it gives the probability of finding phase I at position x . This function tells us the porosity of the bone. We know that if the bone is more porous it is diseased. I used a program written by Nelson Beebe to create these images to create Hashin-Shtrickman coated spheres. The circles are randomly placed in the domain. The parameters I controlled were the maximum number of circles created, the minimum and maximum radii of the circles, and the number of different radii levels. After generating this picture, I converted it to a jpg file and used Matlab to read the image. Matlab "reads" the picture by giving the each pixel a number based on its color. It gives this in the form of a giant matrix. I normalized to give only the numbers one and zero for the colors, where each number refers to a phase. Now, I never got a correlation function for my images, but if I had I would have been able to use this information as a bound to homogenize the structure. Bone is an elastic modulus, and by homogenizing an elastic modulus one can measure and predict the reaction to stress and strain. This would be let us know the fracture risk of a bone. It would also have been interesting to compare the correlation functions of both healthy and diseased bone.