

REU PROJECT PROPOSAL

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Robust Reprojection bases for Chebyshev and Legendre Projections

Recovering a function from its Fourier coefficients or orthogonal polynomial expansion is a task that arises often in scientific and engineering practices, including, for example, medical imaging (tomography), data compression, fluid flow, and image processing . Reconstructing functions that are analytic and periodic over the entire interval in question is well understood; however, for functions which are piecewise analytic, convergence of the reconstruction slows down dramatically and large oscillations are introduced near the discontinuity. Gottlieb et al. [1] have developed a reconstruction method using Gegenbauer polynomials, but this method can be inaccurate and even fail to converge.

Using a method developed by Tanner and Gelb [2], however, one can recover fast convergence from an approximation by using a different reprojection basis. Called a robust Gibbs complementary basis, it imposes an additional requirement– As the number of the original projection increases, the weight function of the orthogonal reprojection basis converges to a weight whose associated orthogonal polynomial basis converges exponentially. In the case of Fourier coefficients, reconstruction bases were derived from Freud weight functions.

With the help of Dr. Jared Tanner, the goal of this project is to extend the above result for Fourier coefficients to Chebyshev, Legendre, and other orthogonal polynomial projections. This includes studying these projections and numerical experimentation to find a set of weights and reconstruction basis, since a theoretical result may be out of reach. Additionally, instead of a fixed truncation of the number of terms in the original basis, one can find the convex hull of given coefficients and stop at the minima. Overall these two results should provide a helpful tool in the post-processing of signals.

Finally, if time permits, an investigation into the accuracy and convergence properties of approximations of functions over an interval vs. subdivisions of that same interval. A result here may provide some insight into H-P refinement of finite element methods.

REFERENCES

- [1] D. Gottlieb and Chi-Wang Shu, On the Gibbs phenomenon and its resolution, SIAM Review 39 (1998), pp. 644-668.
- [2] Anna Gelb and Jared Tanner, Robust reprojection methods for the resolution of the Gibbs phenomenon, Applied and Computational Harmonic Analysis, vol. 20(1) (2006), pp. 3-25.