REU Report

My REU research this semester was spent further investigating composite materials and their effective properties. I considered both critical paths and the imaging of composite materials. Primarily, however, I considered variation principles to define certain characteristics of composite materials. As its name indicates, this technique involves varying parameters to result in bounds for some property. In particular, I looked at the bounds for conductivity of a composite structure derived by integrating over minimal divergence and curl free fields. Namely, the minimal fields that satisfy the following:

$$\nabla \times E = 0, \ \nabla \cdot J = 0$$

Subsequently, the effective conductivity, $\sigma^*$ of a material composed of two different materials of conductivities $\sigma_1, \sigma_2$ in corresponding volume fractions $p_1, p_2$ is bounded as follows:

$$\frac{1}{p_1 \sigma_1 + p_2 \sigma_2} \leq \sigma^* \leq p_1 \sigma_1 + p_2 \sigma_2$$

In addition to studying composite materials, I was able to create a graph using Matlab of the following function (shown on next page) which was used in a grant proposal of Ken Golden's this fall:

$$f(x) = \begin{cases} 
\frac{1}{p \cdot q}, x \in Q, x = \frac{p}{q} \\
0, x \in Q^c 
\end{cases}$$

Variations of this intriguing function are found in quasiperiodic systems. Its interesting behavior may be a basis for investigation of photonic quasicrystals.