

**PROJECT PROPOSAL**  
**REU SUMMER 2005**  
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For the past year I have been studying groups of the form  $F_n \rtimes_f Z$ , ( $F_n$  semi-direct  $Z$ ) where  $f$  is an automorphism of a free group. Over the summer I would like to continue my work on these groups. From the past semesters I read through various articles by Thomas Brady to get a good list of groups  $F_n \rtimes_f Z$ , where  $f$  is an automorphism of a free group of rank three and less, that can act geometrically on a CAT(0) 2-simplex. A group that can act geometrically on a CAT (0) space is said to be a CAT (0) group. This however did not lead to a generalization in the obvious way. It may be true that some of these groups can act geometrically on a CAT (0) 3-simplex. Steve Gersten showed gave an example of  $F_n \rtimes_f Z$  that cannot act geometrically on any CAT (0) space. For the summer, I would like to answer the generalization of Brady's findings: When can  $F_n \rtimes_f Z$ , where  $f$  is an automorphisms of a free group of rank greater than three, act geometrically on a CAT (0) 2-simplex? If  $F_n \rtimes_f Z$  cannot act geometrically on a CAT (0) 2-simplex under what conditions will it have a  $Z^2$  subgroup?

Another property that I have been looking at is the relation between  $\delta$ -hyperbolic and CAT (0) hyperbolic. A space is said to be  $\delta$ -hyperbolic if for every geodesic triangle there exists a  $\delta > 0$  such that the union of a  $\delta$ -neighborhood of two distinct sides contains the third side. We then define what it means for a group to be  $\delta$ -hyperbolic by requiring that its Cayley graph satisfy the  $\delta$ -hyperbolic condition given above. In an article written by Martin Bridson he proves that if  $G$  is  $\delta$ -hyperbolic and acts on a CAT (0) space then they act with no embedded flat planes. Therefore we will have a distinction between groups that are CAT (0) groups and those groups that are  $\delta$ -hyperbolic groups. A question to ask then is: If  $G$  is  $\delta$ -hyperbolic, how do you decide that  $G$  is CAT (0)?

Next Fall, I will be attending the University of Utah as a first year graduate student. I have also received a graduate teaching assistantship for next year. I believe that the past year's REU has been a very valuable part of my education at the University of Utah.