What is the smallest set of numbers such that each of the thirteen powers of 2, from $2^1$ up to and including $2^{13}$, can be obtained by way of multiplying or dividing numbers from the set at most once? (To be clear, no element from the set you give should be used more than once to represent any power of 2 in the thirteen powers of 2.) Show your work and justify your answer.