Consider the triangle $P_1P_2P_3$ and point $P$ within the triangle. The lines $P_1P$, $P_2P$, and $P_3P$ intersect the opposite sides in points $Q_1$, $Q_2$, and $Q_3$, respectively. Prove that, of the numbers
\[
\frac{P_1P}{PQ_1}, \quad \frac{P_2P}{PQ_2}, \quad \frac{P_3P}{PQ_3},
\]
at least one is $\leq 2$ and at least one is $\geq 2$.

In the spirit of UPSC, you should not use the internet or look up the solution in a book. Please include your name, student ID number and email address on your solution.