

Calculus Challenge 2002

1. Find the sum: $\frac{1}{4} + \frac{3}{16} + \frac{5}{64} + \frac{7}{256} + \cdots$
2. Find $\lim_{x \rightarrow \infty} [(x^6 + x^5)^{\frac{1}{6}} - (x^6 - x^5)^{\frac{1}{6}}]$.
3. The horizontal line $y = c$, $c > 0$ intersects the curve $y = 2x - 3x^3$ in the first quadrant as shown. Find c so that the areas of the two shaded regions are equal.
4. Find $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{k^2 + n^2}$
5. Find $\int \frac{dx}{x(x+1)(x+2)+\cdots+(x+m)}$, where m is a positive integer.
6. Let C be the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}}$ where $x \geq 0$ and $y \geq 0$. Find the length of the longest line segment that lies in the first quadrant and is tangent to C .