

### Calculus Challenge 2001

1. Evaluate the integral  $\int_0^{\infty} \frac{dx}{1+x^3}$
2. Show that  $x^e \leq e^x$  for all positive real numbers  $x$  and determine all number  $x$  for which equality holds.
3. Let  $\alpha > -1$  and  $\beta > -1$ . Calculate  $\lim_{n \rightarrow \infty} n^{\beta-\alpha} \frac{1^\alpha + 2^\alpha + \dots + n^\alpha}{1^\beta + 2^\beta + \dots + n^\beta}$
4. Find the sum of the following series:  $\sum_{n=1}^{\infty} \frac{\ln(2^n)}{e^n}$
5. Find the antiderivative:  $\int \frac{dx}{\sqrt{e^x - e^{-x}}}$
6. Does the series  $\sum_{n=1}^{\infty} n e^{-\sqrt{n}}$  converge or diverge? Justify your answer.