Vulgar Finance, Divine Mathematics

Sports Gambling, Financial Derivatives, and Arbitrage

Flip a penny

Heads: you owe me $1

Tails: I owe you $1

Wanna play?

Spin a penny

Heads: you owe me $1

Tails: I owe you $1

Wanna play?

If you spin a penny, it comes up tails roughly 80% of the time.

Why? The heads side is slightly heavier.

We have an edge!...but we can still lose a lot of money...

Let’s Get Ready to Rumble

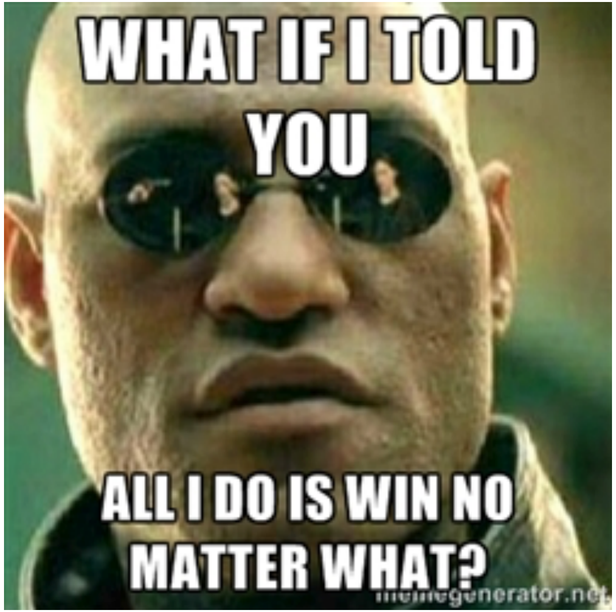
vs

Arbitrage = Free Lunch

It doesn’t matter who we think is going to win; we can lock in a riskless profit by betting the right amounts on BOTH outcomes.

Arbitrage = Free Lunch

Notice that you don't need any money upfront to use this strategy; first, get paid by the person who owes you and with the proceeds, pay off the other person.



Stocks

Represent ownership shares in a company

Options: Calls

Give the owner the RIGHT to BUY a certain amount of STOCK (S) at a specified STRIKE PRICE (K) at a specified EXPIRATION TIME (T)

Example: The May 2016 call with strike price $50 gives the owner the right to buy stock for $50 in May 2016

The value of the call option at expiration is:

$$\mathbf{C(T) = \max\{S(T) - K, 0\}}$$

Options: Puts

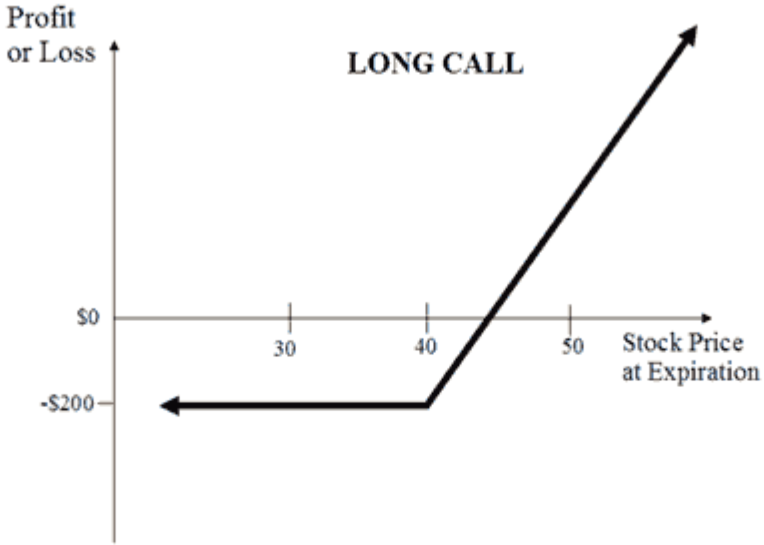
Give the owner the RIGHT to SELL a certain amount of STOCK (S) at a specified STRIKE PRICE (K) at a specified EXPIRATION TIME (T)

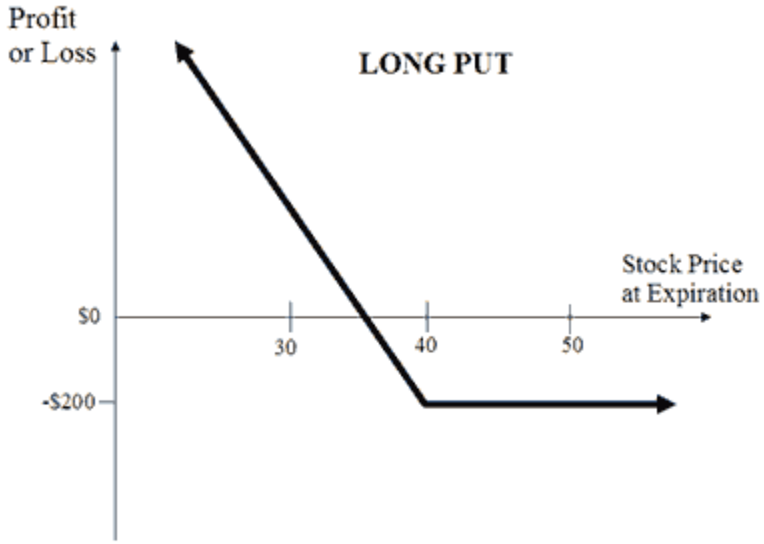
Example: The May 2016 put with strike price $50 gives the owner the right to sell stock for $50 in May 2016

The value of the put option at expiration is:

$$\mathbf{P(T) = \max\{K - S(T), 0\}}$$

Options: Payoff Diagrams





Compound Interest

Suppose the interest rate is r and you want to borrow \$P for T years. At time T, how much will you owe?

Example:

$r = 0.1$ (10%), $P = 100$, $T = 2$

Compounded annually: $P(1 + r)^T = 100(1.1)^2 = 121$

Compounded semi-annually: $P(1 + r/2)^{2T} = 100(1.05)^4 = 121.55$

Compounded quarterly: $P(1 + r/4)^{4T} = 100(1.025)^8 = 121.84$

Compounded continuously: $\lim_{n\to\infty} P(1 + r/n)^{nT} = Pe^{rT} = 122.14$

Two Portfolios

Portfolio 1: Long call, short put with strike K → C - P

Portfolio 2: Long stock, borrow Ke^{-rT} → $S - Ke^{-rT}$

What are the payoffs at time T?

Portfolio 1: $C(T) - P(T) = \max\{S(T) - K, 0\} - \max\{K - S(T), 0\} = S(T) - K$

Portfolio 2: $S(T) - Ke^{-rT}e^{rT} = S(T) - K$

They have the same payoff, so they should cost the same

Arbitrage Part II

Portfolio 1: Long call, short put with strike K

Portfolio 2: Long stock, borrow Ke^{-rT}

What happens if portfolio 1 is cheaper than portfolio 2?

Short portfolio 2, and with the proceeds, go long portfolio 1. This will make you net positive and at time T, the two portfolios will offset each other.

An Example

$K = 50$, $r = 0.1$, $T = 0.25$ (3 months from now)

Stock is trading at \$55

Call costs \$5, put costs \$1

Portfolio 1: Long call, short put: \$5 - \$1 = \$4

Portfolio 2: Long stock, borrow Ke^{-rT}: $\$55 - \$50e^{-(0.1)(0.25)} = \$6.23$

Short portfolio 2 (+\$6.23), long portfolio 1 (-\$4) → net profit of \$2.23