The game of criss cross [1]

Materials: sheet of paper, 2 pencils, 2 hands (any combination of lefts and rights will suffice) belonging to two different people preferably, but not necessarily.

The board: Three points are places at vertices of an (approximately) equilateral triangle. Players choose to put two to seven points in the interior of the said imaginary triangle. It is best if no three points are collinear.

The game: Players alternately draw segments between points on the board so that no two segments intersect. The player who puts in the last legal segment wins the game.
Dramatic introduction:

Hand One "You choose what the board will look like, and I will choose who goes first!" – cloud: " he he, it won't know what hit it".

Hand Two "I suppose that sounds fair." – and proceeds to its demise.

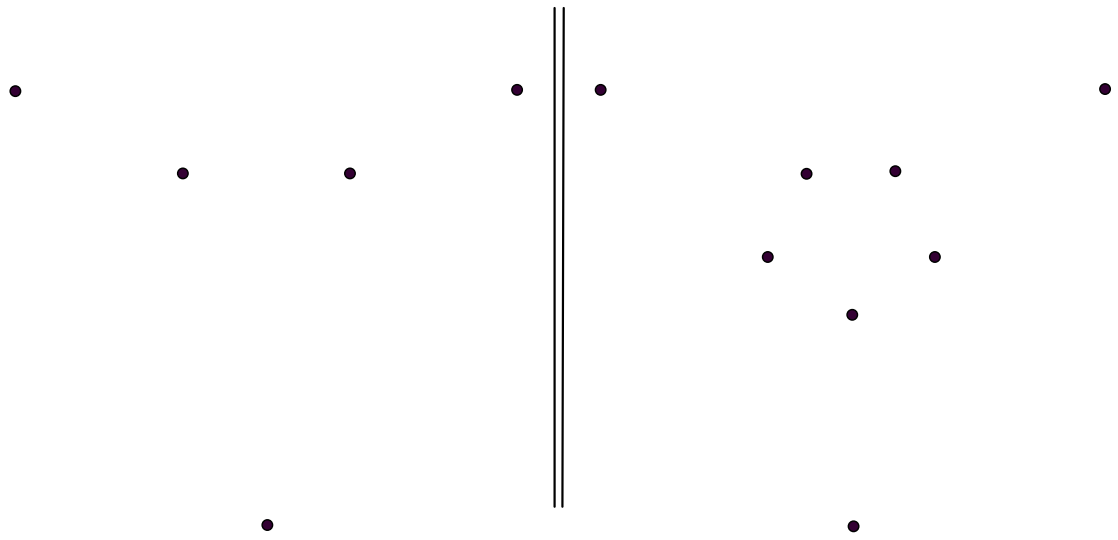

Figure 1: Exemplary boards

1 Play several games with another hand using a left hand board above. How many moves will each game last? Who wins?

2 Do the same for the right hand board. Conjecture the outcome of any such game.

3 What was Hand One all about? Help it explain its own brilliance.

Do not turn the page

[1] Shamelesly and mostly stolen from the book Circle in a Box by Sam Vandervelde. My rendition is funnier.

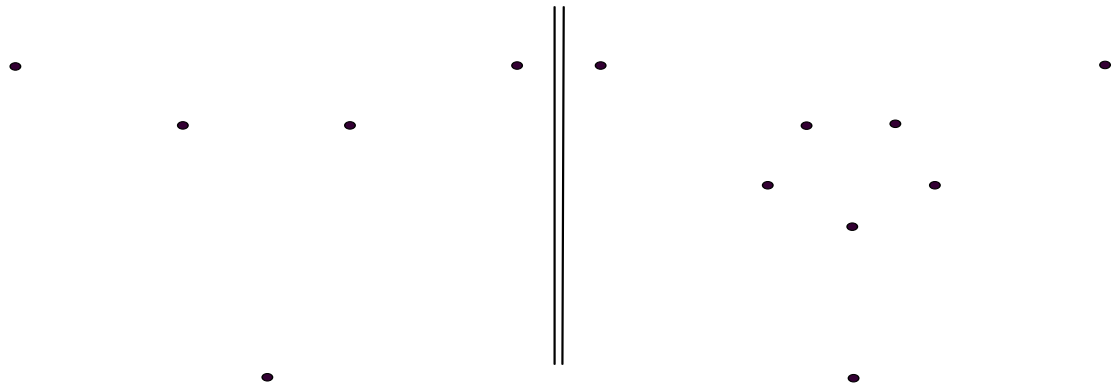

Figure 2: Exemplary boards

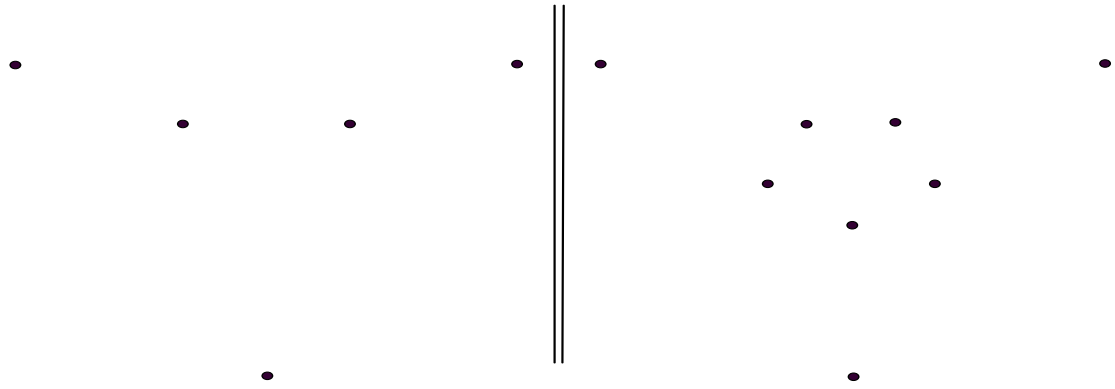

Figure 3: Exemplary boards

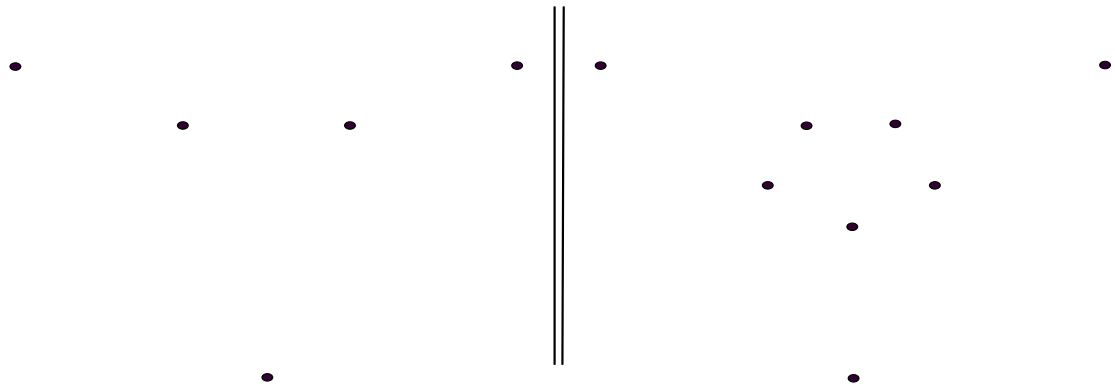

Figure 4: Exemplary boards

HA

4 What would happen if we changed the outer boundary of our board so that it contains four points at the vertices of a square? Can you predict who will win? Prove your conjecture. Extend your result to game board with pentagonal boundaries and beyond.

5 How many pentagons does a football [2] have, and what does that have to do with criss cross?

6 Plot five points on a sheet of paper and draw a segment connecting every pair of points. How many edges are needed? Can you position the points so that none of the edges cross?

[2] No wisecracks about that ball you call football, please!