State Junior Mathematics Contest
Spring 2007

1. How many whole number divisors does the number 2007 have?

    (a) 2 (b) 3 (c) 4 (d) 6 (e) 8

2. $66_{\text{eight}} + 132_{\text{four}} + 1011_{\text{two}} = \underline{\hspace{2cm}}_{\text{eight}}$.

    (a) 101 (b) 135 (c) 137 (d) 1131 (e) 1211

3. The decimal $0.\overline{9} = 0.999\ldots$ is equal to

    (a) 1 (b) $1 - (\frac{9}{10})^{10}$ (c) $(\frac{9}{10})^{\frac{10}{9}}$ (d) 999/1000 (e) 9/10

4. How many whole numbers between 99 and 999 are divisible by 4, 6 and 9?

    (a) 13 (b) 24 (c) 25 (d) 27 (e) 31

5. If $M/5$ has a remainder of 2 and $N/5$ has a remainder of 4, then $(M+N)/5$ will have a remainder of what?

    (a) 0 (b) 1 (c) 2 (d) 3 (e) 5

6. How many children are there in a family in which each boy has as many sisters as brothers but each girl has twice as many brothers as sisters?

 (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

7. Five players are on the Academic Team. Their names do not indicate their gender.

    - Three are girls and two are boys.
    - Two wear white shirts and three wear black shirts.
    - MIKEN and CARRY wear different color shirts.
    - BARI and JAMIE wear the same color shirt.
    - PITA and CARRY are the same gender.
    - JAMIE and MIKEN are different genders.
    - The boy with the white shirt scored the most points.
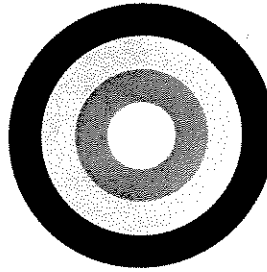
    Who is the leading scorer?

    (a) *BARI* (b) *CARRY* (c) *JAMIE* (d) *MIKEN* (e) *PITA*

8. Find the area of the shaded region. The petals are formed by semicircles and the square is 8 cm on a side.
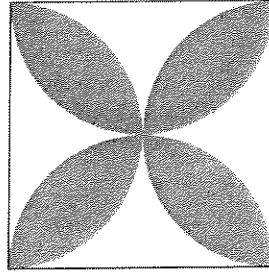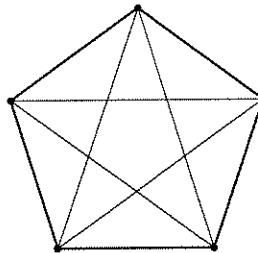

(a) $32(\pi - 2)$ (b) $8(\pi - 1)$ (c) $16(\pi - 1)$ (d) $64 - 32\pi$ (e) $16(\pi - 2)$

9. The radii of the circles on this target are 1, 2, 3, and 4 inches. What is the probability that a random shot that hits the target will hit the bull's eye (i.e., the inner circle)?

(a) $1/4$ (b) $1/8$ (c) $1/16$ (d) $1/4\pi$ (e) $1/2\pi$

10. How many triangles are in this drawing?

(a) 10   (b) 20   (c) 25   (d) 30   (e) 35

11. Solve for x. $|3x - 2| \leq 17$

(a) no solution
(b) $x \geq \frac{19}{3}$ or $x \leq -5$
(c) $x \geq -5$
(d) $x \leq \frac{19}{3}$
(e) $-5 \leq x \leq \frac{19}{3}$

12. Let r be a real number–positive, negative or zero. Which of the following numbers is always greater than r?

(a) $r^2 + 1$ (b) $2r$ (c) $\sqrt{|r|} + \frac{r}{2}$ (d) $(r+1)^3$ (e) r^{100}

13. Ms Trong gets a ten percent raise every year. Her salary after four such raises has gone up by about what percent?

(a) 40% (b) 42% (c) 44% (d) 46% (e) 48%

14. Given this regular hexagon of side $\sqrt{3}$, squares DEFG and CBGH, line ABD, find the area of the triangle ABC.
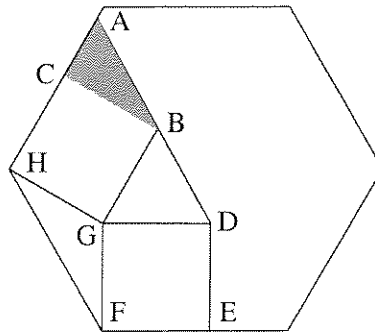
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{\sqrt{3}}{6}$ (e) none

15. For a function defined for all natural numbers by

$$f(n+1) = f(n) + f(n-1),$$

and beginning with $f(1) = 1, f(2) = 1$, for which value of n is $f(n)$ a multiple of 4?

(a) If $n = 3k$, then $f(n)$ is a multiple of 4
(b) If $n = 4k$, then $f(n)$ is a multiple of 4
(c) If $n = 5k$, then $f(n)$ is a multiple of 4
(d) If $n = 6k$, then $f(n)$ is a multiple of 4
(e) It is not possible to predict which terms will be a multiple of 4.

16. If you lose 20% on an investment during the first year and gain 25% the following year, what is your net gain over the two years?

(a) 0% (b) 5% (c) 2.5% (d) -5% (e) 1.25%

17. The number 2^{29} is a 9-digit number with distinct digits. Which digit is missing?

(a) 0 (b) 3 (c) 4 (d) 5 (e) 7

18. If this pattern continues, where would the number 289 apppear?

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            1
        3       5
    7       9       11
13      15      17      19
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(a) 8th element in row 16
(b) 9th element in row 17
(c) 9th element in row 18
(d) last element in row 17
(e) last element in row 18

19. A set of 26 encyclopedias (one for each letter) is placed on a bookshelf in alphabetical order from left to right. Each encyclopedia is 2 inches thick including the front and back covers. Each cover (front or back) is 1/4 inch thick. A bookworm eats straight through the encyclopedias, beginning inside the front cover of volume A and ending after eating through the back cover of volume z. How many inches of book did the bookworm eat?

(a) 48 (b) 48.5 (c) 51.25 (d) 51.5 (e) 51.75

20. Solve for x. $2x^2 - 3x = 9 - 3x^2$

(a) $\frac{3\pm3\sqrt{7}}{10}$

(b) $\frac{-3\pm3\sqrt{21}}{10}$

(c) $\frac{3\pm3\sqrt{21}}{10}$

(d) $\frac{3\pm\sqrt{171}i}{10}$

(e) $\frac{-3\pm3\sqrt{7}}{10}$

21. If w, x, y, z are positive real numbers such that $w + x + y + z = 2$, then

$$N = (w + x)(y + z)$$

satisfies

(a) $0 \leq N \leq 1$
(b) $1 \leq N \leq 2$
(c) $2 \leq N \leq 3$
(d) $3 \leq N \leq 4$
(e) $4 \leq N \leq 5$

22. Triangle ABC has sides 10, 24, and 26 cm long. A rectangle that has an area equal to that of the triangle has width 3 cm. Find the perimeter of the rectangle.

(a) 40 cm   (b) 43 cm   (c) 56 cm   (d) 68 cm   (e) 86 cm

23. $_mP_n = {_{10}C_7}$ for what values of m and n?

(a) $m = 7, n = 10$
(b) $m = 10, n = 3$
(c) $m = 5, n = 4$
(d) $m = 5, n = 1$
(e) $m = 7, n = 5$

24. A recipe calls for $\frac{2}{3}$ of a cup of sugar. You find that you only have $\frac{1}{2}$ cup of sugar left. What fraction of the recipe can you make?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) $\frac{4}{3}$

25. Find the measure of y given the following picture and the fact that $p||n||m$.

(a) 46° (b) 62° (c) 106° (d) 74° (e) 59°
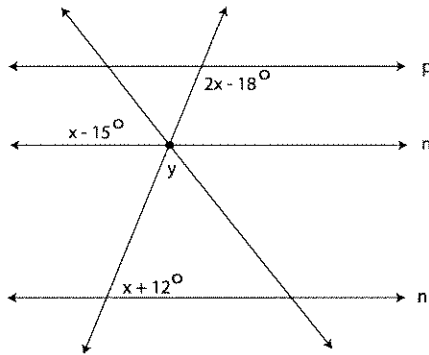
26. In three years, Mary will be three times my present age, and I will be half as old as she. How old is Mary now?

    (*a*) 6 yrs   (*b*) 9 yrs   (*c*) 15 yrs   (*d*) 24 yrs   (*e*) 27 yrs

27. Fred can mow the lawn in 3 hours. Joe can do it in 2 hours. If Fred, Joe and Susan work together to mow the lawn, they can do it in $\frac{12}{13}$ of an hour. How long does it take Susan to mow the lawn herself?

    (*a*) 2 hrs   (*b*) 3 hrs   (*c*) 4 hrs   (*d*) 5 hrs   (*e*) 6 hrs

28. A young man spent $\frac{1}{4}$ of his allowance on a movie. He spent $\frac{11}{18}$ of the remainder on afterschool snacks. Then from the money remaining, he spent $3 on a magazine, which left him with $\frac{1}{24}$ of his allowance to put into savings. How much of his allowance did he save?

    (*a*) $0.50   (*b*) $1.00   (*c*) $1.25   (*d*) $12   (*e*) $31

29. Sally has 4 red flags, 3 green flags and 2 white flags. How many 9-flag signals can she run up a flagpole?

    (*a*) 4!3!2!   (*b*) $\frac{7!}{4}$ (*c*) 9!   (*d*) $\frac{9!}{4!3!}$ (*e*) $\frac{7!}{4!3!}$

30. At a party, 66 handshakes took place. Each person shook hands exactly once with each of the others present. How many people were at the party?

    (*a*) 9   (*b*) 10   (*c*) 11   (*d*) 12   (*e*) 13