

Since τ is a *linear* function, it follows from these equations that

$$\tau = a \left(t - \frac{v}{c^2 - v^2} x' \right)$$

where a is a function $\phi(v)$ at present unknown, and where for brevity it is assumed that at the origin of k , $\tau = 0$, when $t = 0$.

With the help of this result we easily determine the quantities ξ, η, ζ by expressing in equations that light (as required by the principle of the constancy of the velocity of light, in combination with the principle of relativity) is also propagated with velocity c when measured in the moving system. For a ray of light emitted at the time $\tau = 0$ in the direction of the increasing ξ

$$\xi = c\tau \text{ or } \xi = ac \left(t - \frac{v}{c^2 - v^2} x' \right).$$

But the ray moves relatively to the initial point of k , when measured in the stationary system, with the velocity $c - v$, so that

$$\frac{x'}{c - v} = t.$$

If we insert this value of t in the equation for ξ , we obtain

$$\xi = a \frac{c^2}{c^2 - v^2} x'.$$

In an analogous manner we find, by considering rays moving along the two other axes, that

$$\eta = c\tau = ac \left(t - \frac{v}{c^2 - v^2} x' \right)$$

when

$$\frac{y}{\sqrt{c^2 - v^2}} = t, \quad x' = 0.$$

Thus

$$\eta = a \frac{c}{\sqrt{c^2 - v^2}} y \text{ and } \zeta = a \frac{c}{\sqrt{c^2 - v^2}} z.$$

Substituting for x' its value, we obtain

$$\begin{aligned}H_0 - E_0 &= K_0 + C, \\H_1 - E_1 &= K_1 + C\end{aligned}$$

since C does not change during the emission of light. So we have

$$K_0 - K_1 = L \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}.$$

The kinetic energy of the body with respect to (ξ, η, ζ) diminishes as a result of the emission of light, and the amount of diminution is independent of the properties of the body. Moreover, the difference $K_0 - K_1$, like the kinetic energy of the electron (§10), depends on the velocity.

Neglecting magnitudes of fourth and higher orders we may place

$$K_0 - K_1 = \frac{1}{2} \frac{L}{c^2} v^2.$$

From this equation it directly follows that :-

If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2 . The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that

The mass of a body is a measure of its energy-content; if the energy changes by L , the mass changes in the same sense by $L/9 \times 10^{20}$, the energy being measured in ergs, and the mass in grammes.

It is not impossible that with bodies whose energy-content is variable to a high degree (e.g. with radium salts) the theory may be successfully put to the test.

If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies.

1. Find the eigenvalue λ of the matrix $A = \begin{bmatrix} r-3 & 5 & 6 \\ 0 & -1 & 2 \\ -4 & 0 & 1 \end{bmatrix}$.

2. Let $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$, where $n \geq 2$. Show that

$$\begin{vmatrix} 1 & \alpha_1 & \alpha_1^2 & \dots & \alpha_1^{n-1} \\ 1 & \alpha_2 & \alpha_2^2 & \dots & \alpha_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_n & \alpha_n^2 & \dots & \alpha_n^{n-1} \end{vmatrix} = \prod_{1 \leq i < j \leq n} (\alpha_j - \alpha_i).$$

Suggestion: Do it for $n = 2$ and 3 and then try touse induction on n .

3. Derive the following inequalities:

$$\begin{aligned} \sum_{j=1}^{\infty} |\xi_j \eta_j| &\leq \left(\sum_{k=1}^{\infty} |\xi_k|^p \right)^{1/p} \left(\sum_{m=1}^{\infty} |\eta_m|^q \right)^{1/q} \\ \left(\sum_{j=1}^{\infty} |\xi_j + \eta_j|^p \right)^{1/p} &\leq \left(\sum_{k=1}^{\infty} |\xi_k|^p \right)^{1/p} + \left(\sum_{m=1}^{\infty} |\eta_m|^p \right)^{1/p} \end{aligned}$$

4. Given a point $x_0 \in X$ and a real number $r > 0$, we define three types of sets:

$$B(x_0; r) = \{x \in X : d(x, x_0) < r\}$$

$$\tilde{B}(x_0; r) = \{x \in X : d(x, x_0) \leq r\}$$

$$S(x_0; r) = \{x \in X : d(x, x_0) = r\}$$

5.

Day	Min Temp	Max Temp	Summary
Monday	11° C	22° C	A clear day with lots of sunshine. However, the strong breeze will bring down the temperatures.
Tuesday	9°	19°	Cloudy with rain, across many northern regions. Clear spells across most of Scotland and Northern Ireland, but rain reaching the far northwest.