Lab Rotation Presentation

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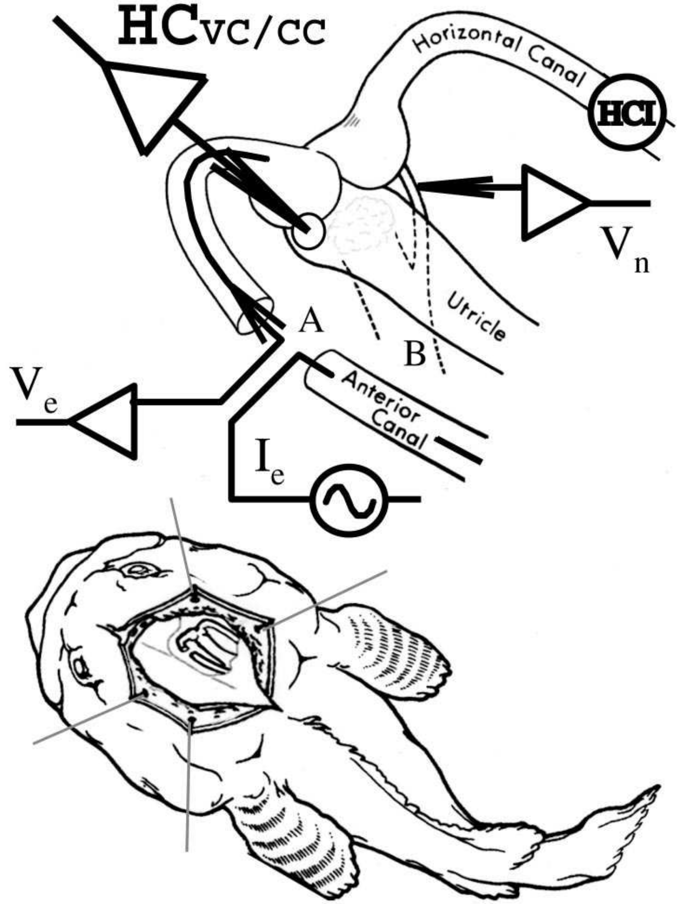
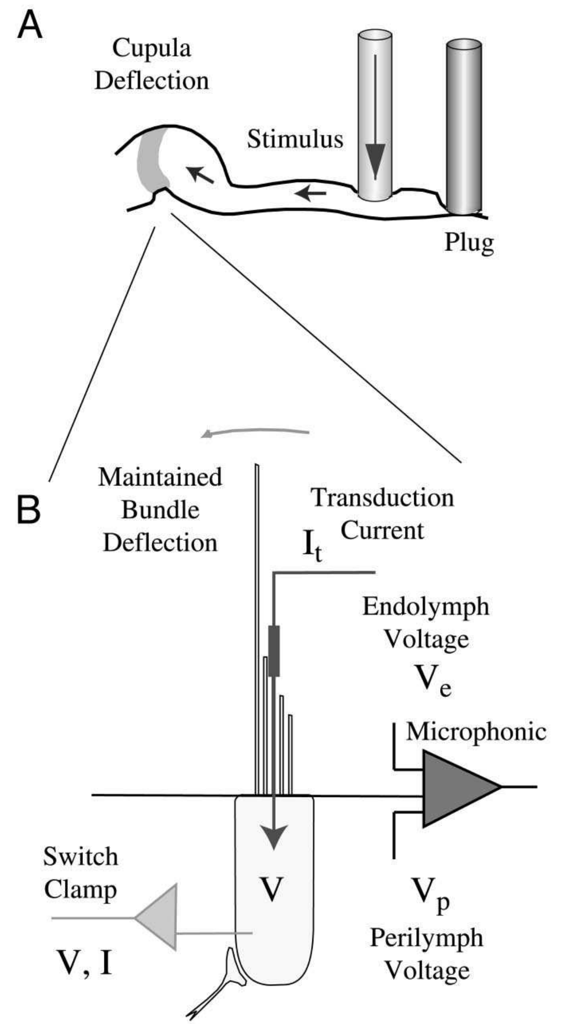
Biology



HCvc/cc
Horizontal Canal
HCI
V_n
Utricle
A
B
V_e
Anterior Canal
I_e

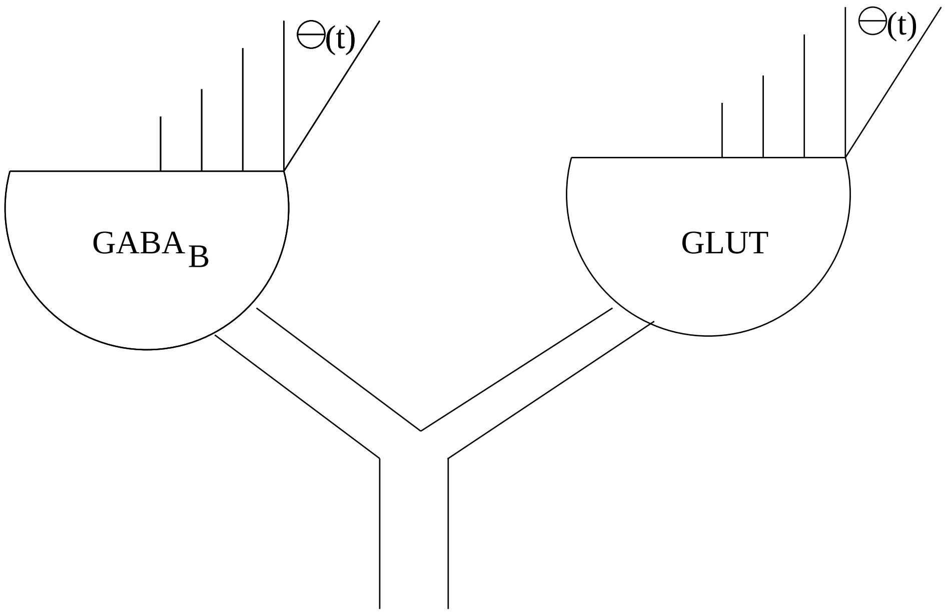
Experimental setup



Temporal encoding

- Low frequency: Fluid displacement $D(t) \propto \ddot{\theta}(t)$
- High frequency: Real time integration via viscous forces in each canal causes $D(t) \propto \dot{\theta}(t)$
- Reflecting this there is a subset of afferents that report $\ddot{\theta}(t)$ for low f and $\dot{\theta}(t)$ for high f
- **However** a substantial number of afferents report $\ddot{\theta}(t)$ or a signal between $\ddot{\theta}(t)$ and $\dot{\theta}(t)$ even for frequencies where $D(t) \propto \dot{\theta}(t)$

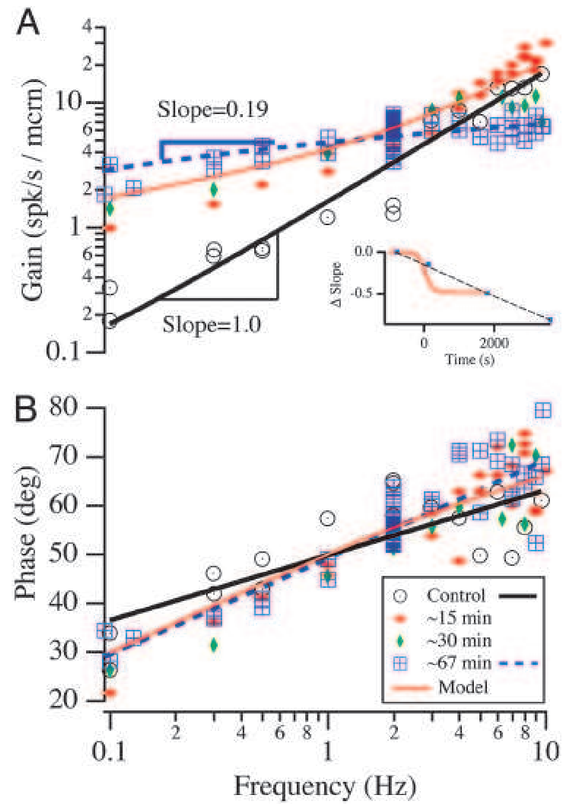
Toy Model



- Transfer function:

$$
\begin{aligned}
Re^{\mathrm{i}\omega t} &= T_E e^{\mathrm{i}\omega t} - T_I e^{\mathrm{i}\omega(t-\delta)} \\
&\approx T_E e^{\mathrm{i}\omega t}(1 - ce^{-\mathrm{i}\omega\delta})
\end{aligned}
$$

Is R a Fractional Derivative?



$R \equiv f(t) - cf(t-\tau) = bD^\alpha f$ for $c, \tau, b, \alpha \neq 0?$

Where $\widehat{D^\alpha f(t)} = (\mathrm{i}s)^\alpha \hat{f}(s)$

No

$$
\begin{aligned}
D^{-\alpha} f(t) &= \frac{1}{\Gamma(\alpha)} \int_l^t (t-s)^{\alpha-1} f(s) ds \\
D^{\alpha} f(t) &= D^m D^{-(m-\alpha)} f(t) \\
(\mathrm{i}s)^{\alpha} \hat{f}(s) &= \widehat{D^{\alpha} f(t)} = (f(t) \widehat{- cf(t} - \tau)) \\
&= (1 - ce^{\mathrm{i}s\tau}) f\hat{(s)}
\end{aligned}
$$

Therefore

$$\forall s \neq 0 \;\; s.t. \;\; \hat{f}(s) \neq 0, \infty$$
$$1 - ce^{\mathrm{i}s\tau} = b(\mathrm{i}s)^{\alpha}$$

Taylor expanding about $a \neq 0$ gives:

$$b(\mathrm{i}a)^{\alpha} = 1 - ce^{(\mathrm{i}a\tau)} \quad O(1) \tag{1}$$
$$\alpha b(\mathrm{i}a)^{\alpha} = -\mathrm{i}ac\tau e^{(\mathrm{i}a\tau)} \quad O(s-a) \tag{2}$$
$$\alpha(\alpha-1)b(\mathrm{i}a)^{\alpha} = a^2c\tau^2e^{(\mathrm{i}a\tau)} \quad O((s-a)^2) \tag{3}$$

Conclusion

(2) and (3) $\Rightarrow$

$$ba^{\alpha-2}\alpha(\alpha-1)/c\tau^2 = \cos(a\tau+\alpha\pi/2) = 0$$

$$\alpha ba^{\alpha-1}/c = sin(a\tau+\alpha\pi/2) = 0$$

$$\Rightarrow\Leftarrow$$

as all parameters are assumed nonzero and $\alpha \neq 1$. Therefore R is not a fractional derivative.