Finite Group Theory

Fletcher Gross

Professor, 1972 B. S. 1960, California Institute of Technology Ph.D. 1964, California Institute of Technology Assistant Professor, Occidental College, 1963-65 Assistant Professor, University of Alberta, 1965-67 Associate Professor, University of Utah, 1967-1972

Activities and Awards

Mathematics Department Undergraduate Teaching Award, 1999

Mathematical Interests

Roughly speaking, Finite Group Theory can be divided into two large problems: (1) the classification problem: find all finite simple groups, and (2) the extension problem: determine the group G if we know all the composition factors of G. My interest has been in the second of these two problems. Initially, I worked on solvable groups and such groups may be regarded as the simplest case in (2) in that the composition factors are cyclic groups of prime order. My motivation for working on solvable groups was with the hope of providing direction for the more general extension problem once the classification problem was solved. Now that the classification problem has been solved, real opportunities exist for carrying this program out and some old problems that resisted solution now appear tractable. In the same way that the determination of the periodic table did not spell the end for Chemistry, the solution of the classification problem has not brought about the end for Finite Group Theory.

In [1] and [2], an old problem of Philip Hall was resolved by showing that if does not include 2, then two Hall -subgroups of a finite group G must be conjugate. The proof depends upon showing that Hall subgroups of a simple group have a very special structure provided that the prime 2 is not involved. This led to a simple method for determining the odd-order Hall subgroups of the classical linear groups which is carried



out in [3].

A joint paper [4] with L. G. Kovács develops methods for general groups, both finite and infinite, that parallel standard methods (group co-homology, for example) used for solvable groups. As examples of results obtained using these methods, we have [5] in which, for example, it is shown that a finite group G has a Hall -subgroup if the automorphism groups of the composition factors of G have Hall -subgroups and [6] in which the problem of determining the automorphism group of a finite group is reduced to finding information about the composition factors of the group. More recently ([7] and [8]), I have applied the results of [6] to determine the automorphism groups of wreath products and groups similar to wreath products.

Selected Publications

1. On a conjecture of Philip Hall, *Proc. London Math. Soc.*, (3) **52** (1986) 464-494.

2. Conjugacy of odd order Hall subgroups, *Bull.* London Math. Soc., **19** (1987) 311-319.

3. Odd order Hall subgroups of the classical linear groups, *Math. Z.* **220** (1995) 317-336.

4. (joint with L. G. Kovács) On normal subgroups which are direct products, *J. Algebra* **90** (1984) 133-168.

5. On the existence of Hall subgroups, J. Algebra 98 (1986), 1-13.

6. Automorphisms of induced extensions, J. Algebra 117 (1988) 457-471.

7. Automorphisms of permutational wreath products, J. Algebra 117 (1988) 472-493.

8. On the uniqueness of wreath products, J. Algebra 147 (1992) 147-175.