WeBWorK assignment number 1.
due 09/09/2008 at 11:59pm MDT.

This first homework serves as an introduction to WeBWorK
and as a review of some relevant preCalculus topics, including:
The language of the number system,
Solution of linear, quadratic, and polynomial equations,
Language of Polynomials
Cartesian Coordinates
Equations and properties of straight lines
logic, converse, contrapositive, and negation of statements
functions, even, odd, graphs, composition
The questions include a few word problems.
Here are some hints on how to use WeBWorK effectively:
After first logging into WeBWorK change your password.
Make sure that at all times WeBWorK has your correct email address.
Find out how to print a hard copy on the computer system
that you are going to use. Contact me if you have any problems.
Print a hard copy of this assignment. Note, however, that the online versions of these problems may have links to the web pages of this course that are absent on the hard copy.

Get to work on each set right away and answer these questions well before the deadline. Not only will this give you the chance to figure out what's wrong if an answer is not accepted, you also will avoid the likely rush and congestion prior to the deadline.

The primary purpose of the WeBWorK assignments in this class is to give you the opportunity to learn by having instant feedback on your active solution of relevant problems. Make the best of it!

Peter Alfeld, JWB 127, 581-6842.

1. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p1.pg
Recall that the natural numbers are
1, 2, 3, 
the integers are
... , −3, −2, −1, 0, 1, 2, 3, 

rational numbers are ratios of integers (with the denominator being non-zero), and real numbers are all numbers corresponding to points on the number line. You can also think of real numbers as repeating or non-repeating decimals. There are more technical definitions which you will learn in real analysis.

Indicate whether the following statements are True (T) or False (F).

1. The difference of two natural numbers is always an integer.
2. The quotient of two natural numbers is always a natural number.
3. The quotient of two natural numbers is always a rational number
4. The product of two natural numbers is always a natural number.
5. The ratio of two natural numbers is always positive
6. The sum of two natural numbers is always a natural number.
7. The difference of two natural numbers is always a natural number.

Correct Answers:
• T
• F
• T
• T
• T
• T
• F

2. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p2.pg
Indicate whether the following statements are True (T) or False (F).

1. The quotient of two integers is always an integer (provided the denominator is non-zero).
2. The difference of two integers is always a natural number.
3. The quotient of two integers is always a rational number (provided the denominator is non-zero).
4. The sum of two integers is always an integer.
5. The ratio of two integers is always positive
6. The difference of two integers is always an integer.
7. The product of two integers is always an integer.

Correct Answers:
• F
• F
• T
• T
• F
• T
• T

3. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p3.pg
Indicate whether the following statements are True (T) or False (F).

1. The difference of two rational numbers is always a rational number.
2. The product of two rational numbers is always a rational number.
3. The quotient of two rational numbers is always a rational number (provided the denominator is non-zero).
4. The sum of two rational numbers is always a rational number.

Correct Answers:
• T
• T
• T
• T
5. The quotient of two rational numbers is always a real number (provided the denominator is non-zero).

6. The ratio of two rational numbers is always positive.

7. The difference of two rational numbers is always a natural number.

**Correct Answers:**
- T
- T
- T
- T
- T
- T
- F
- F

4. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p4.pg

Solve the equation

$$3(x + 2) + 4 = -5(x - 1) - 2$$

for \( x = \) __________.

**Hint:** This is a linear equation.

**Correct Answers:**
- \(-0.875\)

5. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p5.pg

Solve the equation

$$\frac{2}{3-t} + \frac{1}{3+t} + \frac{1}{9-t^2} = 0$$

for \( t = \) __________.

**Hint:** Get rid of the denominators by multiplying with them. Note that the third denominator is a difference of squares.

**Correct Answers:**
- \(-10\)

6. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p6.pg

The equation \(4x^4 - 2x^3 - 3x^2 = 0\) has three real solutions \(A, B,\) and \(C\) where \(A < B < C\)

and \(A = \) __________
and \(B = \) __________
and \(C = \) __________

**Correct Answers:**
- \(-0.651387818865997\)
- 0
- 1.151387818866

7. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p7.pg

Evaluate the expression \(64^{-\frac{4}{3}}\).

(You may enter a fraction as your answer.)

**Correct Answers:**
- 0.00390625

8. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p8.pg

The expression \((3a^4b^5c^2)^2(2a^3b^2c^4)^3\) equals \(n a^r b^s c^t\)

where \(n\), the leading coefficient, is: __________
and \(r\), the exponent of \(a\), is: __________
and \(s\), the exponent of \(b\), is: __________
and finally \(t\), the exponent of \(c\), is: __________

**Correct Answers:**
- 72
- 23
- 16
- 16

9. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p9.pg

Consider the two points \((5, -3)\) and \((9, 10)\). The distance between them is: __________
The \(x\) co-ordinate of the midpoint of the line segment that joins them is: __________
The \(y\) co-ordinate of the midpoint of the line segment that joins them is: __________

**Correct Answers:**
- 13.6014705087354
- 7
- 3.5

10. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p10.pg

Find the distance between \((8, 9)\) and \((-8, -2)\).

**Correct Answers:**
- 19.4164878389476

11. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p11.pg

Find the perimeter of the triangle with the vertices at \((1, 0), (-2, 4),\) and \((-7, -4)\).

**Correct Answers:**
- 23.3782530420558

12. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p12.pg

The equation of the line with slope 5 that goes through the point \((5, 5)\) can be written in the form \(y = mx + b\) where \(m = \) __________
and where \(b = \) __________

**Correct Answers:**
- 5
- -20

13. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p13.pg

This problem is like the preceding problem. The equation of the line with slope 4.5 that goes through the point \((-8.9, 6)\) can be written in the form \(y = mx + b\) where \(m = \) __________
and where \(b = \) __________

**Correct Answers:**
- 4.5
14. (10 pts) The equation of the line that goes through the point (21, 29) and is parallel to the x-axis can be written in the form \( y = mx + b \) where \( m \) is: _____
and where \( b \) is: _____

Correct Answers:
- 0
- 29

15. (10 pts) The equation of the line that goes through the point (3, 9) and is parallel to the line \( 2x + 3y = 3 \) can be written in the form \( y = mx + b \) where \( m \) is: _____
and where \( b \) is: _____

Correct Answers:
- -0.666666666666667
- 11

16. (10 pts) The equation of the line that goes through the point (6, 4) and is perpendicular to the line \( 5x + 2y = 4 \) can be written in the form \( y = mx + b \) where \( m \) is: _____
and where \( b \) is: _____

Correct Answers:
- 0.4
- 1.6

17. (10 pts) A line through (-2, -5) with a slope of -9 has a y-intercept at _____

Correct Answers:
- -23

18. (10 pts) An equation of a line through (-2, 3) which is perpendicular to the line \( y = 4x + 2 \) has slope: _____
and y intercept at: _____

Correct Answers:
- -0.25
- 2.5

19. (10 pts) Find the slope of the line through (8, -1) and (6, 6).

Correct Answers:
- -3.5
25. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p25.pg
Jamestown is 10 miles downstream from Aliceville and on the opposite side of a river half a mile wide. Mary will run from Aliceville along the river for 6 miles, and then swim diagonally (i.e., along a straight line) across the river to Jamestown. If she runs at 8 miles per hour and swims at three miles per hour she will reach Jamestown after _________ hours. Assume that the current is negligible.

Note that unless otherwise specified WeBWorK expects your answer to be within "one tenth of one percent" of the "true answer" which is the answer it has been given by the author of the problem. Practically speaking this means you should specify at least 4 digits total (including any before the decimal point). To be safe, you want to compute your answer with your calculator much more accurately.

Hint: Draw a picture and apply the Pythagorean Theorem;
Correct Answers:
- 2.09370962471642

26. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p27.pg
The next few problems are simple exercises in logic. Consider the statement "A implies B". The converse of this statement is "B implies A", its contrapositive is "not B implies not A", and its negation is "A and not B". For example, consider the statement all natural numbers are real numbers. This is a true statement, but this is actually not important for this discussion. This statement can be put as an implication: if x is a natural number, then x is a real number. The converse of this statement is all real numbers are natural numbers, or if x is a real number then it is a natural number, (which is a false statement), the contrapositive is if x is not a real number then it isn’t a natural number (which is a true statement), and the negation of the statement is some natural numbers are not real numbers (which is a false statement).

The purpose this particular problem is to illustrate the pattern of the next couple of problems. You already know the answers, so it is just a matter of entering them. In this problem, WeBWorK will tell you separately for each answer whether it is correct or not, but in the next two you will have to enter everything correctly to get credit.

Let S be the statement: All natural numbers are real numbers.

Complete the sentences below, filling in A-G from the following list, and T or F for true or false, as appropriate.

A. Numbers aren’t natural.
B. All real numbers are natural.
C. Some natural numbers aren’t real numbers.
D. Some real numbers aren’t natural numbers.
E. No real number is natural.
F. A number can’t be natural if it isn’t real.
G. A number can’t be real if it isn’t natural.

S is ___ (true or false).
The converse of S is ___ (enter a letter from A-G) and that statement is ___.
The contrapositive of S is ___ and that statement is ___.
The negation of S is ___ and that statement is ___(true or false).
Correct Answers:
- T
- B
- F
- F
- T
- C
- F

27. (10 pts) set1/1210s1p28.pg
Let S be the statement: All women are humans.

Complete the sentences below, filling in A-G from the following list, and T or F for true or false, as appropriate.

A. Some women aren’t humans.
B. Some women are humans.
C. Some humans aren’t women.
D. No women are humans.
E. If you aren’t a human you aren’t a woman.
F. Men aren’t human.
G. All humans are women.
$S$ is ___ (true or false).

The converse of "$S$" is ___ (enter a letter from A-G) and that statement is ___ (true or false).

The contrapositive of $S$ is ___ and that statement is ___.

The negation of $S$ is ___ and that statement is ___.

There are two versions of this problem with the roles of "men" versus "women" randomly interchanged.

Correct Answers:
• T
• G
• F
• E
• T
• A
• F

28. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p29.png
Throughout this problem, suppose that $x$ stands for a real number.

Let $S$ be the statement: if $x > 0$ then $x^3 > 0$.

Complete the sentences below, filling in A-G from the following list, and T or F for true or false, as appropriate.

A. Real numbers are all zero.

B. If $x^3 > 0$, then $x > 0$.

C. There is some $x > 0$ such that $x^3 \leq 0$

D. If $x \geq 0$ then $x^3 \leq 0$.

E. If $x > 0$ then $x^3 > x$.

F. $x^3 > x$.

G. If $x^3 \leq 0$ then $x \leq 0$.

$S$ is ___ (true or false).

The converse of $S$ is ___ (enter a letter from A-G) and that statement is ___ (true or false).

29. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s1p30.png
This problem addresses some common algebraic errors. For the equalities stated below assume that $x$ and $y$ stand for real numbers. Assume that any denominators are non-zero. Mark the equalities with T (true) if they are true for all values of $x$ and $y$, and F (false) otherwise.

$$(x + y)^2 = x^2 + y^2.$$  
$$(x + y)^2 = x^2 + 2xy + y^2.$$  
$$\frac{x}{x+y} = \frac{1}{y}.$$  
$$x - (x + y) = y.$$  
$$\sqrt{x^2} = x.$$  
$$\sqrt{x^2} = |x|.$$  
$$\sqrt{x^2 + 4} = x + 2.$$  
$$\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}.$$  

Correct Answers:
• F
• T
• F
• T
• F
• T
• F

30. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p1.png
Let 
$$f(x) = mx + b$$
(where $m \neq 0$). The graph of $f$ is a straight line with slope _____.
Any line perpendicular to that graph has a slope _____.

The graph of the inverse function of $f$ is a straight line with slope _____.

Correct Answers:
• $m$
• $-1/m$
• $1/m$

31. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p2.png
In this question you will derive a general formula for the distance of a point from a line.

Let $P$ be the point $(p,q)$ and $L$ the line $y = mx + b$.

The slope of $L$ is ____________
The slope of a line perpendicular to $L$ is _____________.

The line through $P$ perpendicular to $L$ can be written as $y = sx + c$
where $s$ is: ________________
and $c$ is: _________________.
That line intersects $L$ in the point $Q = (u, v)$,
where $u$ is: ________________
and $v$ is: _________________.
The distance of $P$ and $Q$ is _________________.
(Note, all answers must be in terms of $m$, $b$, $p$ and $q$.)

**Hint:** If you are bewildered by all the symbols ask yourself what they mean in the special cases of the preceding home work, and compare your calculations for this problem with the numerical calculations you did earlier.

**Correct Answers:**
- $m$
- $-1/m$
- $-1/m$
- $q+p/m$
- $(m*(q+p/m - b))/(1+m*m)$
- $m*(m*(q+p/m - b))/(1+m*m) + b$
- $sqrt((q-m*p-b)**2/(1+m**2))$
- $• m*(m*(q+p/m - b))/(1+m*m)$
- $• -1/m$
- $• m$
- $• m*n$
- $• m+n$
- $• m+n$

32. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p3.png
This problem is about polynomial degrees. Recall that the degree of a polynomial $p$ is $n$ if $p$ can be written as

$$p(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$$

where the leading coefficient $a_n \neq 0$.

The degree of $p(x) = 1$ is _____________.
The degree of $p(x) = x^2 + 2x + 3$ is _____________.
The degree of $p(x) = ((x+2)^2(x-1)^3)^2$ is _____________.

Let $p$ and $q$ be polynomials of degree $m$ and $n$, respectively.
The degree of the product of $p$ and $q$ is _____________.
and the degree of the composition of $p$ and $q$ is _____________.

**Hint:** The product of two polynomials $p$ and $q$ is

$$h(x) = p(x) \times q(x)$$
and the composition of $p$ and $q$ is

$$h(x) = p(q(x))$$
or

$$h(x) = q(p(x)).$$
(The two compositions are distinct but they have the same degree. If you are still confused look at simple examples.

**Correct Answers:**
- $0$
- $2$
- $10$
- $m+n$
- $m+n$

33. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p4.png
The next few problems concern even and odd functions. Recall that a function $f$ is even if

$$f(x) = f(-x)$$
for all $x$ in its domain, and it is odd if

$$f(x) = -f(-x)$$
for all $x$ in its domain. The graph of an even functions is symmetric with respect to the $y$-axis, and an odd function is symmetric with respect to the origin. This is an example of one of our major themes: the interplay between algebra and geometry.

For each of the following functions enter "E" to indicate that the function is even, "O" to indicate it is odd, and "N" to indicate that is neither even nor odd.

1. $f(x) = x^3 + x^9 + x^7$
2. $f(x) = x^9 - 6x^{10} + 3x^2$
3. $f(x) = x^{-6}$
4. $f(x) = x^6 + 3x^{10} + 2x^7$

**Correct Answers:**
- $O$
- $E$
- $E$
- $N$

34. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p5.png
Oddly, there is a function that is both even and odd. It is $f(x) =$

**Correct Answers:**
- $0$

35. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p6.png
For each of the following functions enter "E" to indicate that the function is even, "O" to indicate it is odd, and "N" to indicate that is neither even nor odd.

1. $f(x) = \sin x.$
2. $f(x) = \cos x.$
3. $f(x) = \tan x.$

**Correct Answers:**
- $O$
- $E$
- $O$
36. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p9.pg
For each of the following functions enter “E” to indicate that the function is even, “O” to indicate that it is odd, and "N" to indicate that is neither even nor odd.

- \( f(x) = \sin^2 x \).
- \( f(x) = \sin x^2 \).
- \( f(x) = \sin(\cos x) \).
- \( f(x) = \sin(\sin x) \).
- \( f(x) = \sin x + \cos x \).

Correct Answers:
- E
- E
- O
- N

37. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p8.pg
Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false. You must get all of the answers correct to receive credit.

1. The ratio of two odd functions is odd
2. The sum of two even functions is even
3. The composition of an odd function and an odd function is even
4. The product of two odd function is odd
5. The product of two even function is even
6. The sum of an even and an odd function is usually neither even or odd, but it may be even.
7. A function cannot be both even and odd.
8. The composition of an even and an odd function is even

All of the answers must be correct before you get credit for the problem.

Correct Answers:
- F
- T
- T
- F
- T
- T
- T
- T
- T

38. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p9.pg
The next few problems concern the composition of functions, and the fact that this is different from the multiplication of functions. If \( f \) and \( g \) are two functions then their product \( fg \) is defined by

\[ (fg)(x) = f(x)g(x). \]
As usual, the absence of an arithmetic operator denotes multiplication.

The composition \( f \circ g \) of these two functions is quite different. We evaluate first one, and then the other. Thus

\[ (f \circ g)(x) = f(g(x)). \]

Let \( f(x) = 3x + 5 \) and \( g(x) = 5x^2 + 3x. \)
\( (fg)(3) = \) __________

Correct Answers:
- 756

39. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p10.pg
Let \( f(x) = 4x + 6 \) and \( g(x) = 4x^2 + 8x. \) Then
\( (fg)(x) = \) __________

Correct Answers:
- \( 16x^3 + 32x^2 + 48x \)

40. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p11.pg
Let \( f(x) = 6x + 4 \) and \( g(x) = 6x^2 + 6x. \)
Then,
\( (f \circ g)(2) = \) __________

Correct Answers:
- 220

41. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p12.pg
Let \( f(x) = 3x + 3 \) and \( g(x) = 3x^2 + 3x. \)
Then,
\( (f \circ g)(x) = \) __________

Correct Answers:
- \( 9x^2 + 9x + 3 \)

42. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p13.pg
Let \( f(x) = x + 1 \) and \( g(x) = \frac{1}{x+7} \). Then

\[ (f \circ f)(x) = \]
\[ (f \circ g)(x) = \]
\[ (g \circ f)(x) = \]
\[ (g \circ g)(x) = \]

Correct Answers:
- \( x+2 \)
- \( (x+2)/(x+1) \)
- \( 1/(x+2) \)
- \( (x+1)/(x+2) \)

43. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p14.pg
Let \( g(x) = x + 1 \) and \( h(x) = (f \circ g)(x) = x^2 + 2x + 1. \)
Then
\( f(x) = \) __________, and
\( (g \circ f)(x) = \) __________

Correct Answers:
- \( x^2 \)
- \( x^2 + 1 \)
44. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p15.pg

Let \( g(x) = x^2 \) and \( h(x) = (f \circ g)(x) = \sin x^2 \).

Then
\[
\begin{align*}
f(x) &= \underline{\text{__________________}}, \\
(g \circ f)(x) &= \underline{\text{__________________}}.
\end{align*}
\]

**Correct Answers:**
- \( \sin(x) \)
- \( \sin(x)^2 \)

45. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p16.pg

The next few questions provide another variation of the interplay between algebra and geometry. Simple algebraic modifications have simple effects on the graph. Adding to \( x \) shifts the graph left or right, adding to \( y \) shifts it up or down, multiplying \( x \) rescales it horizontally, and multiplying \( y \) rescales it vertically. These effects can of course be combined.

Relative to the graph of 
\[
y = x^2
\]

the graphs of the following equations have been changed in what way?

- 1. \( y = x^2 - 10 \)
- 2. \( y = x^2 + 10 \)
- 3. \( y = (x/10)^2 \)
- 4. \( y = (10x)^2 \)

A. stretched horizontally by the factor 10  
B. shifted 10 units up  
C. shifted 10 units down  
D. compressed horizontally by the factor 10

**Correct Answers:**
- C
- B
- A
- D

46. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p17.pg

Relative to the graph of 
\[
y = \sin(x)
\]

the graphs of the following equations have been changed in what way?

- 1. \( y = \sin(x) - 9 \)
- 2. \( y = \sin(x + 9) \)
- 3. \( y = \sin(x)/9 \)
- 4. \( y = \sin(9x) \)

**Correct Answers:**
- C  
- D  
- B  
- A

47. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p18.pg

Let \( f(x) = x^2 + \sin x \) and let \( g(x) = \underline{\text{__________________}} \), where \( g \) is the function whose graph has been obtained from that of \( f \) by shifting it 5 to the right and 8 up.

**Correct Answers:**
- (x-5)^2 + \sin(x-5) + 8

48. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p19.pg

The function \( f(x) = x^2 + 8x + 23 \) can be obtained from an even function \( g \) by shifting its graph horizontally and vertically. That even function is \( g(x) = \underline{\text{__________________}} \).

Its graph has been shifted by _____ to the left and _____ up.

**Hint:** Complete the Square.

**Correct Answers:**
- \( x^2 \)
- 4
- 7

49. (10 pts) 1250Library/set1_Reviews_of_Fundamentals/1210s2p20.pg

Relative to the graph of 
\[
y = \frac{1}{x^2}
\]

the graphs of the following equations have been changed in what way?

- 1. \( y = \frac{1}{(x-14)^2} \)
- 2. \( y = \frac{1}{x^2} - 14 \)
- 3. \( y = \frac{1}{14^2} \)
- 4. \( y = \frac{1}{196x^2} \)

A. compressed horizontally by the factor 14  
B. compressed vertically by the factor 14  
C. shifted 14 units right  
D. shifted 14 units down

**Correct Answers:**
- C  
- D  
- B  
- A
Math 1250-2 Fall 2008

Hsiang-Ping Huang.

WeBWorK assignment number 2.

due 09/16/2008 at 11:59pm MDT.

This problem set is on limits and derivatives, and some applications.

Peter Alfeld, JWB 127, 581-6842.

1. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p21.pg
   Evaluate the limit
   \[ \lim_{x \to 1} \frac{x^2 + 5x - 6}{x - 1} = \]
   Correct Answers:
   • 0.142857142857143

2. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p22.pg
   Evaluate the limit
   \[ \lim_{x \to 1} \frac{x^3 - x}{x^2 - 1} = \]
   Correct Answers:
   • 1

3. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p23.pg
   Evaluate the limit
   \[ \lim_{b \to 6} \frac{36 - b}{6 - \sqrt{b}} = \]
   Correct Answers:
   • 12

4. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p24.pg
   Evaluate the limit
   \[ \lim_{x \to 6} \frac{x - 6}{x^2 - 9} = \]
   Correct Answers:
   • -0.0277777777777778

5. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p25.pg
   Evaluate the limit
   \[ \lim_{x \to -3} \frac{7x^2 - 7x + 7}{x - 5} = \]
   Correct Answers:
   • -11.375

6. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p26.pg
   \[ \lim_{h \to 0} \frac{(x + h)^3 - x^3}{h} = \]
   (Your answer will be a mathematical expression in \(x\).)
   Correct Answers:
   • 3x^2

7. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p27.pg
   Let
   \[ f(x) = x^2 - 2x + 5. \]
   Then
   \[ \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \]
   Correct Answers:
   • 2x - 2

8. (10 pts) 1250Library/set2_Derivatives_and_Limits/1210s2p28.pg
   Let
   \[ f(x) = 8x^2 - 5x + 5. \]
   Then
   \[ \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \]
   Correct Answers:
   • 2 * 8 * x - 5

9. (10 pts) set2/1210s2p29.pg
   Recall that
   \[ \lim_{x \to c} f(x) = L \]
   means:
   
   For all \( \varepsilon > 0 \) there is a \( \delta > 0 \) such that for all \( x \) satisfying
   \( 0 < |x - c| < \delta \) we have that \( |f(x) - L| < \varepsilon \).

What if the limit does not equal \( L \)? Think about what that means
in \( \varepsilon, \delta \) language.

Consider the following phrases:

1. \( \varepsilon > 0 \)
2. \( \delta > 0 \)
3. \( 0 < |x - c| < \delta \)
4. \( |f(x) - L| > \varepsilon \)
5. but
6. such that for all
7. there is some
8. there is some \( x \) such that

Order these statements so that they form a rigorous assertion that
\[ \lim_{x \to c} f(x) \neq L \]
and enter their reference numbers in the appropriate sequence in
these boxes:

Correct Answers:
• 7
• 1
• 6
• 2
• 8
• 3
10. (10 pts) This exercise will help you review some basic trigonometry on a right triangle. Recall that, by the Pythagorean Theorem, a triangle with sides $a$, $b$ and $c$ has a right angle opposite the (longest) side $c$ if and only if $a^2 + b^2 = c^2$.

This is certainly true for the triangle in this problem. If necessary, review the definitions of the trigonometric functions and their inverses to solve this problem.

Consider the familiar right triangle with sides of lengths 3, 4, and 5 feet. Let $A$ be the angle opposite the side of length 3, and $B$ the angle opposite the side of length 4 feet.

$A = \boxed{\text{radians}} = \boxed{\text{degrees}}$, and $B = \boxed{\text{radians}} = \boxed{\text{degrees}}$.

Correct Answers:
- $0.643501108793284$
- $36.869897645844$
- $0.927295218001612$
- $53.130102354156$

11. (10 pts) Many times in this class we will solve quadratic equations. You may be used to applying the quadratic formula, and that’s fine, but of course the variables $a$, $b$, and $c$ may occur in your quadratic equations in different ways than they are used in the quadratic formula. This problem illustrates the translation from formula to problem, that may be necessary.

There are two solutions of the equation $bx^2 - cx + a^2 = 0$ (where $a$, $b$, and $c$ are constants, and $x$ is the unknown). They differ by the sign of the square root. Enter the one with the plus sign here.

Correct Answers:
- $(c+\sqrt{c^2-4*a*b})/(2*b)$

12. (10 pts) Evaluate $\lim_{x \to -3} x + 2$.

Correct Answers:
- $0.8$

13. (10 pts) $\lim_{x \to 5} \frac{x^2 - 13x + 40}{x - 5} = \boxed{\text{}}$.

Correct Answers:
- $-3$

14. (10 pts) $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x - 2} = \boxed{\text{}}$.

Correct Answers:
- $-1$

15. (10 pts) $\lim_{x \to 4} \frac{x - 4}{\sqrt{x - 2}} = \boxed{\text{}}$.

Correct Answers:
- $4$

16. (10 pts) $\lim_{x \to \infty} \frac{x - 4}{x + 4} = \boxed{\text{}}$.

$\lim_{x \to \infty} \frac{x^2 + 4}{x - 4} = \boxed{\text{}}$.

Correct Answers:
- $1$
- $0$

17. (10 pts) If you toss a rock at an initial height $H$ with an initial velocity $V$ then its height $h(t)$ after $t$ seconds is given by the formula

$$h(t) = -\frac{1}{2}gt^2 + Vt + H$$

where $g = 32$ feet second$^{-2}$ on Earth. (On other celestial bodies $g$ would be different. The minus sign is due to the convention that the positive direction is up and gravity pulls down.)

You have probably seen this formula before, and chances are you were simply told that this is the way it is. With Calculus, we can make perfect sense of this formula. The underlying observation is the experimentally observed fact that, ignoring air resistance, on Earth a free falling object increases its speed by 32 feet per second every second. The velocity of the object is the derivative of height, and the acceleration is the derivative of velocity. So we have to work out that the formula given above has the right properties.

If $h$ is given by the above expression, then $h(0) = \boxed{\text{}}$.

The velocity $v(t)$ is $v(t) = h'(t) = \boxed{\text{}}$ and $v(0) = \boxed{\text{}}$.

Moreover, the acceleration is $v'(t) = \boxed{\text{}}$.

Correct Answers:
- $\boxed{\text{}}$

The point $P(3, 19)$ lies on the curve $y = x^2 + x + 7$. If $Q$ is the point $(x, x^2 + x + 7)$, find the slope of the secant line $PQ$ for the following values of $x$.
If $x = 3.01$, the slope of $PQ$ is: ____________
and if $x = 3.1$, the slope of $PQ$ is: ____________
and if $x = 2.99$, the slope of $PQ$ is: ____________
and if $x = 2.9$, the slope of $PQ$ is: ____________
Based on the above results, guess the slope of the tangent line to the curve at $P(3, 19)$. ____________

**Correct Answers:**
- 7.1
- 7.01
- 6.9
- 6.99
- 7

If a ball is thrown straight up into the air with an initial velocity of $50$ ft/s, its height in feet after $t$ seconds is given by $y = 50t - 16t^2$. Find the average velocity for the time period beginning when $t = 1$ and lasting
(i) 0.5 seconds ____________
(ii) 0.1 seconds ____________
(iii) 0.01 seconds ____________

Finally based on the above results, guess what the instantaneous velocity of the ball is when $t = 1$. ____________

**Correct Answers:**
- 10
- 16.4
- 17.84
- 18

This is a simple exercise in computing derivatives of polynomials. The derivative of

$p(x) = 8x^2 + 3x + 4$

is $p'(x) = ____________$. The derivative of

$q(x) = 7x^5 - 2x^4 + 8x^3 - 5x^2 + 7x + 8$

is $q'(x) = ____________$. 

**Correct Answers:**
- $2*8 \times x + 3$
- $5*7*x^4 - 4*2 * x^3 + 3*8* x^2 - 2*5 x + 7$

The derivative of $p(x) = (2x - 1)^2$ is $p'(x) = ____________$. The derivative of $q(x) = (2x - 1)^3$ is $q'(x) = ____________$. 

**Hint:** As we will learn soon, there are more advanced ways of doing this, but you can compute the derivative by expanding the given expressions and writing each of $p$ and $q$ in the standard form of a polynomial.

**Correct Answers:**
- $4*(2x-1)$
- $3*(2x-1)*x*2*2$

If $f(x) = 2 + 4x - 2x^2$ then $f'(-8)=____________$. 

**Correct Answers:**
- 4

If $f(x) = 3x^2 - 2x - 35$, then $f'(x) =$ ____________ and $f'(1)=____________$. 

**Correct Answers:**
- $2*3*x-2$
- 4

Let $f(x) = \frac{1}{x+3}$ Use the limit definition of the derivative on page 107 to find
(i) $f'(-7) =$ ____________
(ii) $f'(-5) =$ ____________
(iii) $f'(-2) =$ ____________
(iv) $f'(1) =$ ____________

To avoid calculating four separate limits, I suggest that you evaluate the derivative at the point when $x = a$. Once you have the derivative, you can just plug in those four values for "a" to get the answers.

**Correct Answers:**
- -0.0625
- -0.25
- -1
This problem will help you practice computing tangents in the next problem. Let \( f(x) = x^2 \).

Then \( f'(x) = \ldots \).

The tangent to the graph of \( f \) through the point \((1, 1)\) has the slope \ldots and the y-intercept \ldots. It intercepts the x-axis at \( x = \ldots \).

Correct Answers:
- \( 2x \)
- \( 2 \)
- \(-1 \)
- \( 0.5 \)

The slope of the tangent line to the parabola \( y = 2x^2 + 5x + 4 \) at the point \((4, 56)\) is: \ldots

The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \ldots \) and where \( b = \ldots \).

Correct Answers:
- \( 21 \)
- \( 21 \)
- \(-28 \)

The slope of the tangent line to the curve \( y = 4x^3 \) at the point \((-2, -32)\) is: \ldots

The equation of this tangent line can be written in the form \( y = mx + b \) where \( m = \ldots \) and where \( b = \ldots \).

Correct Answers:
- \( 48 \)
- \( 48 \)
- \( 64 \)

This problem is our first introduction to Newton's Method for the solution of nonlinear equations.

The positive solution of \( f(x) = x^2 - 2 = 0 \)

is obviously \( x = \sqrt{2} \).

How might one calculate a numerical value of \( \sqrt{2} \)? The idea of Newton's method is to pick a guess \( x_0 \), compute the tangent to the graph of \( f \) at \((x_0, f(x_0))\), and then replace the guess \( x_0 \) with \( x_1 \) which is the \( x \) intercept of the tangent. (You should draw a picture of this idea.)

Suppose \( x_0 = 1.5 \). The tangent to the graph of \( f \) at the point \((x_0, f(x_0))\) can be written in slope intercept form as \( y = \ldots x + \ldots \). The tangent intercepts the x-axis at \( x_1 = \ldots \).

Now let's repeat the process. The tangent to the graph of \( f \) at the point \((x_1, f(x_1))\) can be written in slope intercept form as \( y = \ldots x + \ldots \). The tangent intercepts the x-axis at \( x_2 = \ldots \). Note the accuracy of this approximation: \( \sqrt{2} - x_2 = \ldots \).

Note: To obtain the same result in your last answer as we you need to compute all intermediate answers to the full accuracy of your calculator. To accomplish this store all intermediate results in your calculator. Do not copy them on paper and then reenter them by hand later. Doing so introduces rounding errors and compromises the accuracy of your calculations and solutions. In fact, you should make a habit of this every time you use your calculator: store intermediate results and avoid having to reenter them by hand later. Doing so introduces rounding errors and compromises the accuracy of your calculations and solutions.

The tangent intercepts the x-axis at \( x_1 = \ldots \).

The tangent intercepts the x-axis at \( x_2 = \ldots \).

Note the accuracy of this approximation: \( \sqrt{2} - x_2 = \ldots \).

Note: To obtain the same result in your last answer as we you need to compute all intermediate answers to the full accuracy of your calculator. To accomplish this store all intermediate results in your calculator. Do not copy them on paper and then reenter them by hand later. Doing so introduces rounding errors and compromises the accuracy of your calculations and solutions. In fact, you should make a habit of this every time you use your calculator: store intermediate results and avoid having to reenter them by hand later. Doing so introduces rounding errors and compromises the accuracy of your calculations and solutions.

The particle stops moving (i.e. is in a rest) twice, once when \( t = A \) and again when \( t = B \) where \( A < B \). \( A \) is \ldots and \( B \) is \ldots.

What is the position of the particle at time 16?

Finally, what is the TOTAL distance the particle travels between time 0 and time 16? 

Correct Answers:
- 90
- 3
- 5
- 3488
- 3504
You toss a rock up vertically at an initial speed of \( v \) feet per second and release it at an initial height of \( H \) feet. The rock will remain in the air for ______ seconds. It will reach a maximum height of ______ feet after ______ seconds.

Note: Ignore air resistance.

**Correct Answers:**
- \( \frac{v}{g} \)
- \( \frac{\sqrt{v^2 + 2gh}}{g} \)
- \( \frac{v^2}{2g} + H \)
- \( 0.8125 \)
- \( 16.5625 \)
- \( 1.82992628725623 \)

You shoot a rifle at an angle of 46 degrees. The bullet leaves your rifle at a height of 6 feet and a speed of 854 feet per second. The bullet leaves the bow at a speed of ______ miles per hour.

**Note:** My son and I did worry about the effect of air resistance. The arrows were sticking out of the sand where they had hit the beach. So my son shot another arrow downwards at an angle of 45 degrees straight into the sand. It penetrated to about the same depth as the arrows that had flown the distance. So they must have hit at about the same speed. We concluded that ignoring air resistance in this case was reasonable.

**Hint:** The speed of the arrow is measured in the direction in which it flies. The significance of shooting at an angle of 45 degrees is that initially the horizontal and the vertical components of the speed are equal. There are 3600 seconds in an hour and 5280 feet in a mile.

**Correct Answers:**
- \( 94.475498594666 \)

You shoot an arrow at an angle of 45 degrees. It hits the ground at a distance of \( d \) feet. The arrow left the bow at a speed of ______ miles per hour.

**Hint:** The speed of the arrow is measured in the direction in which it flies. The significance of shooting at an angle of 45 degrees is that initially the horizontal and the vertical components of the speed are equal. There are 3600 seconds in an hour and 5280 feet in a mile.

**Correct Answers:**
- \( \frac{\sqrt{32d}}{5280 \times 3600} \)

If a ball is thrown vertically upward from the roof of a 64 foot tall building with a velocity of 80 ft/sec, its height after \( t \) seconds is \( s(t) = 64 + 80t - 16t^2 \). What is the maximum height the ball reaches? ______

**What is the velocity of the ball when it hits the ground (height 0)? ______**
This fun little question was motivated by a discussion I had with students after class. Consider the absurd statement:

All numbers are equal.

Figure out what is wrong with the proof below and email me your explanation. I will forward the best explanations to the class, with your name attached.

**Proof:** Let \( n \) be a natural number. Let \( S_n \) be the statement that in a set of \( n \) numbers all numbers are equal. I shall prove by induction that \( S_n \) is true for all natural numbers \( n = 1, 2, 3, \ldots \).

Clearly, \( S_1 \) is true. If a set contains just one number then all numbers in that set are equal.

I now need to show that the truth of \( S_k \) implies the truth of \( S_{k+1} \). (Note: In class I used \( n \) instead of \( k \). I m hoping that \( k \) is clearer. Using \( k \) instead of \( n \) is not the problem.) Let

\[ M = \{a_1, a_2, \ldots, a_{k+1}\} \]

be a set of \( k + 1 \) numbers. Then the subset

\[ M_1 = \{a_1, a_2, \ldots, a_k\} \]

of \( M \) contains \( k \) numbers, and by the induction hypothesis these are all equal. Similarly, the subset

\[ M_2 = \{a_2, a_3, \ldots, a_{k+1}\} \]

of \( M \) contains \( k \) numbers all of which are equal. In other words, we have

\[
\begin{align*}
  a_1 &= a_2 &= \ldots &= a_k \\
  a_2 &= \ldots &= a_k &= a_{k+1}
\end{align*}
\]

Together these equations imply that

\[
\begin{align*}
  a_1 &= a_2 &= \ldots &= a_k &= a_{k+1}
\end{align*}
\]

Thus all the numbers in \( M \) are equal. QED

To get credit for this ww question, type the sentence "I read this", without the quotation marks, here: _______.

**Correct Answers:**

- Ireadthis
1. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p5.pg

If

\[ f(x) = (x^2 + 5x + 2)^4 \]

then

\[ f'(x) = \text{____________________________} \]

Correct Answers:

- \((4(x^2+5x+2)^{4-1})*(2x+5)\)

2. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p7.pg

If

\[ f(x) = (x^3 + 2x + 2)^2 \]

then

\[ f'(x) = \text{____________________________} \]

Correct Answers:

- \((2(x^3+2x+2)^{2-1})*(3x^2+2)\)

3. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p8.pg

If

\[ f(x) = (3x + 7)^{-2} \]

then

\[ f'(x) = \text{____________________________} \]

and

\[ f'(3) = \text{____________________________} \]

Correct Answers:

- \((-2*(3x+7)^{-2-1})*(3)\)
- \(-0.00146484375\)

4. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p11.pg

If

\[ f(x) = \sqrt{2x+5} \]

then

\[ f'(x) = \text{____________________________} \]

Correct Answers:

- \((0.5*(2x+5)^{0.5}*(-0.5))*2\)

5. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p13.pg

If

\[ f(x) = \sqrt{4x^2 + 4x + 3} \]

then

\[ f'(x) = \text{____________________________} \]

and

\[ f'(1) = \text{____________________________} \]

Correct Answers:

- \((0.5*(4*x*x+4*x+3)^{-0.5}*(-0.5))*(2*x+1)\)
- \(1.80906806746658\)

6. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p19.pg

If

\[ f(x) = \frac{8x^4}{1-x} \]

then

\[ f^{(4)}(x) = \text{____________________________} \]

Note: There is a way of doing this problem without using the quotient rule 4 times.

Correct Answers:

- \(8*24*(1-x)^{-1-4}\)

7. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p20.pg

A space traveler is moving from left to right along the curve

\[ y = x^2. \]

When she shuts off the engines, she will continue traveling along the tangent line at the point where she is at that time. At what point

\[ (x,y) = (\text{______, ______}) \]

should she shut off the engines in order to reach the point \((4,15)\)?

If she was traveling from right to left she would have to shut off the engines at the point

\[ (x,y) = (\text{______, ______}). \]

Correct Answers:

- 3
- 9
- 5
- 25
As discussed in class (and in the textbook in section 11.4), to solve the equation
\[ f(x) = 0 \]
by Newton’s Method we start with a good initial guess \( x_0 \) and then run the iteration
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots \]
until we get an approximation \( x_{n+1} \) that is good enough for our purposes.

Suppose you want to compute the cube root of 4 by solving the equation
\[ x^3 - 4 = 0. \]
Since \( 1^3 = 1 \) and \( 2^3 = 8 \) Let’s start with \( x_0 = 1.5 \)

Then
\[ x_1 = \ldots, \]
\[ x_2 = \ldots, \]
\[ x_3 = \ldots, \] and
To check your answer compute \( x_3^3 = \ldots. \)

Enter \( x_1, x_2 \) and \( x_3 \) with at least 6 correct digits beyond the decimal point.

Correct Answers:
- 1.59259259259259
- 1.58741795698709
- 1.587401052

If we start Newton’s Method with \( x_0 \) being close to one of these solutions we will get convergence to that solution. On the other hand, note that
\[ f'(x) = 3x^2 - 1 \]
is zero when
\[ x = \pm \frac{1}{\sqrt{3}}. \]

Thus the tangent will be horizontal in those two cases, and Newton’s can’t even be carried out.

In this problem we’ll investigate what happens in the contrived case that
\[ x_0 = \frac{\sqrt{5}}{5}. \]

You can enter \( x_0 \) into WeBWorK as \( \sqrt{5}/5 \). Try it:
\[ x_0 = \ldots. \]

Now do your computations using exact arithmetic, and you’ll recognize a pattern:
\[ x_1 = \ldots, \]
\[ x_2 = \ldots, \]
\[ x_3 = \ldots, \]
\[ x_4 = \ldots. \]

Draw a picture to see what’s going on. Note, however, that Newton’s method may fail in many different ways. A detailed analysis of Newton’s method and related methods is a huge subject and well beyond the scope of our class.

Correct Answers:
- 0.447213595499958
- -0.447213595499958
- 0.447213595499958
- -0.447213595499958
- 0.447213595499958

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The equation
\[ 10(x - 1)(x - 2)(x - 3) = 1 \]
has three real solutions
\[ a < b < c \]
where
\[ a = \ldots, \]
\[ b = \ldots, \] and
\[ c = \ldots. \]

Enter your answers with at least six correct digits beyond the decimal point.

Hint: Ask what the solutions are if the right hand side is 0 instead of 1, and use Newton’s Method.

Correct Answers:
- 1.05435072607641
- 1.89896874211899
- 3.0466805318046
As discussed in class, there may be more variables floating around than the one with respect to which we differentiate. In that situation it is important to be aware with respect to which variable we differentiate.

For example, the volume of a square box width \( s \) and height \( h \) is given

\[ V = s^2 h. \]

Thus

\[ D_s V = \quad \text{and} \quad D_h V = \quad. \]

Correct Answers:

- \( 2s \cdot h \)
- \( s \cdot s \)

12. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p26.pg

The total resistance \( R \) of two parallel resistors \( S \) and \( T \) is given by

\[ R = \frac{ST}{S + T}. \]

Thus

\[ D_S R = \quad \text{and} \quad D_T R = \quad. \]

Correct Answers:

- \( T \cdot S^2 / (S + T)^2 \)
- \( S \cdot T^2 / (S + T)^2 \)

13. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p27.pg

This problem is in preparation for the next two problems. Suppose you have a cube of length \( s \). The volume of that cube is

\[ V = s^3. \]

Now let’s suppose the dimensions of that cube (and hence its volume) depend on time. We are wondering about the relationship between the growth of the length versus the growth of the volume.

Suppose

\[ s(t) = t. \]

Then \( s'(t) = \quad \text{and} \quad V'(t) = \quad. \)

Next, suppose

\[ V(t) = t. \]

Then \( s'(t) = \quad \text{and} \quad V'(t) = \quad. \)

Correct Answers:

- \( 1 \)
- \( 3 \cdot t \cdot t \)
- \( t \cdot (2/3) / 3 \)

14. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p28.pg

The radius of a spherical watermelon is growing at a constant rate of 2 centimeters per week. The thickness of the rind is always one tenth of the radius. The volume of the rind is growing at the rate \( \quad \) cubic centimeters per week at the end of the fifth week. Assume that the radius is initially zero.

Correct Answers:

- \( 681.097287298267 \)

15. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s4p29.pg

Your neighbor is growing a slightly different watermelon. It also has a rind whose thickness is one tenth of the radius of that watermelon. However, the rind of your neighbor’s watermelon grows at a constant rate of 20 cubic centimeters a week. The radius of your neighbor’s watermelon after 5 weeks is \( \quad \) and at that time it is growing at \( \quad \) centimeters per week.

Correct Answers:

- \( 4.4952891419823 \)
- \( 0.296635260946549 \)

16. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s5p1.pg

Let

\[ f(x) = x(x^2 + 4)^{15}. \]

\[ f'(x) = \quad \text{and} \quad f''(x) = \quad. \]

Correct Answers:

- \( (31 \cdot x^2 + 4) \cdot (x^2 + 4)^{14} \)
- \( 30 \cdot (31 \cdot x^2 + 12) \cdot (x^2 + 4)^{13} \cdot x \)

17. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-/1210s5p6.pg

Compute

\[ D_1 x = \quad, \]
\[ D_2 x^2 = \quad, \]
\[ D_3 x^3 = \quad, \]
\[ D_4 x^4 = \quad, \]
\[ D_5 x^5 = \quad, \]
\[ D_6 x^6 = \quad. \]

Do you recognize a pattern?

Compute \( D_6^2 (2x^6 + 3x^5 + 4x^4 + 5x^3 + 17x^2 - 15x + \pi) = \quad \) and \( D_5^3 (2x + 1)^5 = \quad. \)

Correct Answers:

- \( 1 \)
- \( 2 \)
- \( 6 \)
- \( 24 \)
- \( 120 \)
- \( 720 \)
- \( 1440 \)
- \( 3840 \)
Consider these statements written in ordinary language:

A. The speed of the car is proportional to the distance it has traveled.

B. The car is speeding up.

C. The car is slowing down.

D. The car always travels the same distance in the same time interval.

E. We are driving backwards.

F. Our acceleration is decreasing.

Denoting by $s(t)$ the distance covered by the car at time $t$, and letting $k$ denote a constant, match these statements with the following mathematical statements by entering the letters A through E on the appropriate boxes:

- $s'' < 0$
- $s'$ is constant
- $s'' > 0$
- $s'' < 0$
- $s' = k$

Correct Answers:

- C
- D
- E
- F
- A

Find the slope of the tangent line to the curve

$$-2x^2 - 4xy - 3y^3 = -1440$$

at the point $(-4, 8)$.

Your answer: __________

Hint: Differentiate implicitly.

Correct Answers:

- $-0.0285714285714286$

Suppose $3x^2 + 5x + xy = 5$ and $y(5) = -19$. Find $y'(5)$ by implicit differentiation.

Your answer: __________

Hint: You’ll also have to solve for $y$.

Correct Answers:

- $-3.2$

Let $A$ be the area of a circle with radius $r$. If $\frac{dr}{dt} = 2$, find $\frac{dA}{dt}$ when $r = 3$.

Your answer: __________

Correct Answers:

- $37.6991118$

Use implicit differentiation to find the equation of the tangent line to the curve $xy^3 + xy = 14$ at the point $(7, 1)$. The equation of this tangent line can be written in the form $y = mx + b$ where $m$ is: _____

and where $b$ is: _____

Correct Answers:

- $-0.0714285714285714$
- $1.5$

Suppose $\frac{r^2}{36} + \frac{x^2}{36} = 1$ and $y(2) = 5.65685$. Find $y'(2)$ by implicit differentiation.

Correct Answers:

- $-0.353553390593274$

Find the slope of the tangent line to the curve (a lemniscate)

$$2(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point $(3, 1)$.

$m = $ __________

Correct Answers:

- $-0.692307692307692$
25. (10 pts) Suppose $\sqrt{x} + \sqrt{y} = 11$ and $y(9) = 64$. Find $y'(9)$ by implicit differentiation.

**Correct Answers:**
- $-2.66666666666667$

26. (10 pts) Let $xy = 4$ and let $\frac{dy}{dt} = 1$ Find $\frac{dx}{dt}$ when $x = 1$.

**Correct Answers:**
- $-0.25$

27. (10 pts) The graph of the equation

$$x^2 + xy + y^2 = 9$$

is a slanted ellipse. Think of $y$ as a function of $x$. Differentiating implicitly and solving for $y'$ gives: $y' = \ldots$. (Your answer will depend on $x$ and $y$.)

The ellipse has two horizontal tangents. The upper one has the equation $y = \ldots$.

The right most vertical tangent has the equation $x = \ldots$.

That tangent touches the ellipse where $y = \ldots$.

**Hint:** The horizontal tangent is of course characterized by $y' = 0$. To find the vertical tangent use symmetry, or think of $x$ as a function of $y$, differentiate implicitly, solve for $x'$ and then set $x' = 0$.

**Correct Answers:**
- $-(2x+y)/(x+2y)$
- $3.46410161513775$
- $3.46410161513775$
- $-1.73205080756888$

28. (10 pts) A street light is at the top of a 12.0 ft. tall pole. A man 5.6 ft tall walks away from the pole with a speed of 3.5 feet/sec along a straight path. How fast is the tip of his shadow moving when he is 32 feet from the pole?

**Your answer:**

**Hint:** Draw a picture and use similar triangles.

**Correct Answers:**
- $6.5623$

29. (10 pts) The altitude of a triangle is increasing at a rate of 2.5 centimeters/minute while the area of the triangle is increasing at a rate of 5.0 square centimeters/minute. At what rate is the base of the triangle changing when the altitude is 8.5 centimeters and the area is 98.0 square centimeters?

**Your answer:**

**Hint:** The area $A$ of a triangle with base $b$ and height $h$ is given by $A = \frac{1}{2}bh$.

**Differentiate implicitly.**

**Correct Answers:**
- $-5.60553633217993$

30. (10 pts) Gravel is being dumped from a conveyor belt at a rate of 30 cubic feet per minute. It forms a pile in the shape of a right circular cone whose base diameter and height are always equal to each other. How fast is the height of the pile increasing when the pile is 20 feet high? Recall that the volume of a right circular cone with height $h$ and radius of the base $r$ is given by $V = \frac{1}{3}\pi r^2h$.

**Your answer:**

**Correct Answers:**
- $0.0954929658551372$

31. (10 pts) Water is leaking out of an inverted conical tank at a rate of 10900 cubic centimeters per minute at the same time that water is being pumped into the tank at a constant rate. The tank has height 7 meters and the diameter at the top is 6.5 meters. If the water level is rising at a rate of 18 centimeters per minute when the height of the water is 1.0 meters, find the rate at which water is being pumped into the tank in cubic centimeters per minute.

**Your answer:**

**Correct Answers:**
- $132796.99856735$
32. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-1210s5p23.pg
A spherical snowball is melting in such a way that its diameter is decreasing at rate of 0.2 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 18 cm.
Your answer ________ (cubic centimeters per minute) should be a positive number.

**Hint:** The volume of a sphere of radius $r$ is

$$V = \frac{4\pi r^3}{3}.$$  

The diameter is twice the radius.

**Correct Answers:**

- 101.78760186

33. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-1210s5p24.pg
You are blowing air into a spherical balloon at a rate of $\frac{4\pi}{3}$ cubic inches per second. (The reason for this strange looking rate is that it will simplify your algebra a little.) Assume the radius of your balloon is zero at time zero. Let $r(t), A(t),$ and $V(t)$ denote the radius, the surface area, and the volume of your balloon at time $t,$ respectively. (Assume the thickness of the skin is zero.)

All of your answers below are expressions in $t$:

$r'(t) =$ ________ inches per second,

$A'(t) =$ ________ square inches per second, and

$V'(t) =$ ________ cubic inches per second.

**Hint:** The surface area $A$ and the volume $V$ of a sphere of radius $r$ are given by

$$A = 4\pi r^2 \text{ and } V = \frac{4\pi r^3}{3}.$$  

**Correct Answers:**

- $t^{(-2/3)}/3$
- $8*3.14159265358979/3*t^{(-1/3)}$
- $4.18879020478639$

34. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-1210s5p25.pg
You may want to solve this problem only after you solve the next one, which is the general version of this one. On the other hand, you also may want to solve this one first, and when solving the general one compare your evolving solution against your results in this problem.

A child is flying a kite. If the kite is 150 feet above the child’s hand level and the wind is blowing it on a horizontal course at $v$ feet per second, the child is paying out cord at ________ feet per second when 210 feet of cord are out. Assume that the cord remains straight from hand to kite.

**Correct Answers:**

- $2.79941684889506$

35. (10 pts) 1250Library/set3_Rates_of_Change_and_the_Chain_Rule-1210s5p26.pg
You say goodbye to your friend at the intersection of two perpendicular roads. At time $t = 0$ you drive off North at a (constant) speed $v$ and your friend drives West at a (constant) speed $w$. You badly want to know: how fast is the distance between you and your friend increasing at time $t$? Enter here the derivative of the distance from your friend with respect to $t$:

Being scientifically minded you ask yourself how does the speed of separation change with time. In other words, what is the second derivative of the distance between you and your friend?

Suppose that after your friend takes off (at time $t = 0$) you linger for an hour to contemplate the spot on which he or she was standing. After that hour you drive off too (to the North). How fast is the distance between you and your friend increasing at time $t$ (greater than one hour)?

Again, you ask what is the second derivative of your separation:

If you wish e-mail me your comments on how lingering makes things harder, mathematically speaking.

**Correct Answers:**

- $\sqrt{v^2+w^2}$
- 0
- $(v^2*(t-1)+w^2*t)/\sqrt{(v^2*(t-1))^2+w^2*t^2}$
- $(v^2*w^2)/(\sqrt{(t-1)^2+v^2+w^2*t^2}*(t-1)^2+v^2+w^2*t^2)$

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Math 1250-2 Fall 2008
Hsiang-Ping Huang.

WeBWorK assignment number 4.
due 09/30/2008 at 11:59pm MDT.

This problem set covers our class work during the week of September 15.

1. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s6p3.pg

\[ \frac{d}{dx}(ax^2 + bx + c) = \frac{d}{dx}(ax^2 + bx + c) = \frac{d}{dx}(2ax + b) \]

Correct Answers:
- \( 2ax + b \)

2. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s6p4.pg

Suppose \( f(t) = \sqrt{t^2 + 1} \).

Then

\[ f'(t) = \frac{d}{dt}(\sqrt{t^2 + 1}) \]

\[ f''(t) = \frac{d}{dt}(f'(t)) \]

\[ f'''(t) = \frac{d}{dt}(f''(t)) \]

Correct Answers:
- \( \frac{t}{\sqrt{t^2 + 1}} \)
- \( \frac{1}{\sqrt{t^2 + 1}}(1 + t^2) \)
- \( \frac{1}{(t^2 + 1)^2}(t + 2t^3) \)

3. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s6p5.pg

Suppose you want to compute the fifth root of 6 by solving the equation

\[ f(x) = x^5 - 6 = 0 \quad (*) \]

using Newton’s method. Newton’s method starts with an initial approximation \( x_0 \) and then computes a sequence of approximations \( x_1, x_2, x_3, \ldots \) via the formula

\[ x_{k+1} = g(x_k), \quad k = 0, 1, 2, \ldots \]

where

\[ g(x) = x - \frac{f(x)}{f'(x)} \]

For the function defined above in (*), \( g(x) = \ldots \)

Letting

\[ x_0 = 1 \]

you obtain

\[ x_1 = \ldots \]

\[ x_2 = \ldots \]

\[ x_3 = \ldots \]

Correct Answers:
- \( x - \frac{(x^5 - 6)}{(5x^4)} \)
- 2
- 1.675
- 1.49244809457499


For what values of \( x \) does the graph of

\[ f(x) = 2x^3 - 9x^2 + 6x + 0 \]

have a horizontal tangent? Enter the \( x \) values in order, smallest first, to 4 places of accuracy:

\[ x_1 = \ldots < x_2 = \ldots \]

Correct Answers:
- 2
- 1.675
- 1.49244809457499


The function

\[ f(x) = -4x^3 - 30x^2 + 72x - 2 \]

is increasing on the interval \( (-\infty, \ldots) \) and the interval \( \ldots, \infty) \).

The function has a local maximum at \( \ldots \).

Correct Answers:
- \( -6 \)
- \( 1 \)
- \( -6 \)
- \( 1 \)
- \( 1 \)


Find the point on the line \( 4x + 7y - 7 = 0 \) which is closest to the point \((-4, -2)\).

\( (\ldots, \ldots) \)

Correct Answers:
- \(-1.72307692307692 \)
- \( 1.98461538461538 \)


A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola \( y = 7 - x^2 \). What are the dimensions of such a rectangle with the greatest possible area?

Width = \ldots Height = \ldots

Correct Answers:
- 3.05505046330389
- 4.66666666666667
A cylinder is inscribed in a right circular cone of height 7 and radius (at the base) equal to 6. What are the dimensions of such a cylinder which has maximum volume?
Radius = 4
Height = 2.33333333333333
Correct Answers:
- 4
- 2.33333333333333

A fence 6 feet tall runs parallel to a tall building at a distance of 2 feet from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building? 10.811197454847
Correct Answers:
- 10.811197454847

If 1600 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
Volume = 6158.40287135601 cubic centimeters.
Correct Answers:
- 6158.40287135601

The function \( f(x) = 4x + 9x^{-1} \) has one local minimum and one local maximum. It is helpful to make a rough sketch of the graph to see what is happening.
This function has a local minimum at \( x \) equals \(-1.5\) with value \(-12\) and a local maximum at \( x \) equals \(1\) with value \(12\).
Correct Answers:
- \(-1.5\)
- \(12\)
- \(-12\)

A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 26 feet? 47.3283784955167
Correct Answers:
- 47.3283784955167

A rancher wants to fence in an area of 2000000 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use? 6928.2032027551
Correct Answers:
- 6928.2032027551

A University of Rochester student decided to depart from Earth after his graduation to find work on Mars. Before building a shuttle, he conducted careful calculations. A model for the velocity of the shuttle, from liftoff at \( t = 0 \) s until the solid rocket boosters were jettisoned at \( t = 52.7 \) s, is given by
\[
v(t) = 0.001341833t^3 - 0.081295t^2 + 34.76t + 0.25
\]
(in feet per second). Using this model, estimate the absolute maximum value \(37.371487895\) and absolute minimum value \(33.1182469196373\) of the acceleration of the shuttle between liftoff and the jettisoning of the boosters.
Correct Answers:
- \(37.371487895\)
- \(33.1182469196373\)

15. (10 pts) set4/1210s6p19.pg
You are going to make many cylindrical cans. The cans will hold different volumes. But you’d like them all to be such that the amount of sheet metal used for the cans is as small as possible, subject to the can holding the specific volume. How do you choose the ratio of diameter to height of the can? Assume that the thickness of the wall, top, and bottom of the can is everywhere the same, and that you can ignore the material needed for example to join the top to the wall.
Put differently, you ask what ratio of diameter to height will minimize the area of a cylinder with a given volume?
That ratio equals \(1\).
Correct Answers:
- \(1\)

It takes a certain power \( P \) to keep a plane moving along at a speed \( v \). The power needs to overcome air drag which increases as the speed increases, and it needs to keep the plane in the air which gets harder as the speed decreases. So assume the power required is given by
\[
P = cv^2 + \frac{d}{v^2}
\]
where \( c \) and \( d \) are positive constants. (They depend on your plane, your altitude, and the weather, among other things.) Enter here the choice of \( v \) that will minimize the power required to keep moving at speed \( v \).

Suppose you have a certain amount of fuel and the fuel flow required to deliver a certain power is proportional to that power. What is the speed \( v \) that will maximize your range (i.e., the distance you can travel at that speed before your fuel runs out)? Enter your speed here.

Finally, enter here the ratio of the speed that maximizes the distance and the speed that minimizes the required power.

**Correct Answers:**
- \( \frac{d}{c}^{1/4} \)
- \( \frac{3d}{c}^{1/4} \)
- 1.31607401295249

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**17.** (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s6p21.pg

This is a related rates problem with a twist.

Suppose you have a street light at a height \( H \). You drop a rock vertically so that it hits the ground at a distance \( d \) from the street light. Denote the height of the rock by \( h \). The shadow of the rock moves along the ground. Let \( s \) denote the distance of the shadow from the point where the rock impacts the ground. Of course, \( s \) and \( h \) are both functions of time. To enter your answer into WeBWorK use the notation \( v \) to denote \( h' \):

\[ v = h'. \]

Then the speed of the shadow at any time while the rock is in the air is given by \( s' = \) \[ \text{expression} \] (where \( s' \) is an expression depending on \( h, s, H, \) and \( v \) (You will find that \( d \) drops out of your calculation.)

Now consider the time at which the rock hits the ground. At that time

\[ h = s = 0. \]

The speed of the shadow at that time is \( s' = \) \[ \text{expression} \] where your answer is an expression depending on \( H, v, \) and \( d \).

**Hint:** Use similar triangles and implicit differentiation. For the second part of the problem you will need to compute a limit.

**Correct Answers:**
- \( v \cdot s \cdot H/h/|H-h| \)
- \( v \cdot d/H \)

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**18.** (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s7p1.pg

This differentiation problem is a little sneaky. Let

\[ f(x) = \frac{x^2 - 1}{x + 1}. \]

Then

\[ f'(x) = \] \[ \text{expression} \] and

\[ f''(x) = \]

**Hint:** Begin by canceling common factors in numerator and denominator.

**Correct Answers:**
- 1
- 0

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**19.** (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s7p4.pg

Consider the function

\[ f(x) = -2x^3 - 4x^2 + 3x - 2 \]

Find the average slope of this function on the interval \((1,2)\).

By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((1,2)\) such that \( f'(c) \) is equal to this mean slope. Find the value of \( c \) in the interval which works:

**Correct Answers:**
- -23
- 1.51914617476733

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**20.** (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s7p5.pg

Answer the following questions for the function

\[ f(x) = x\sqrt{x^2 + 4} \]

defined on the interval \([-5,7]\).

A. \( f(x) \) is concave down on the interval \[ \text{to} \]

B. \( f(x) \) is concave up on the interval \[ \text{to} \]

C. The inflection point for this function is at \( x = \)

D. The minimum for this function occurs at \( x = \)

E. The maximum for this function occurs at \( x = \)

**Correct Answers:**
- -5
- 0
- 7
- 0
- -5
- 7

The function \( f(x) = 2x^3 - 36x^2 + 120x + 1 \) has one local minimum and one local maximum. It is helpful to make a rough sketch of the graph to see what is happening.

This function has a local minimum at \( x = \) _____ with value \( f(x) = \) ________, and a local maximum at \( x = \) _____ with value \( f(x) = \) ________.

Correct Answers:
- 10
- -399
- 2
- 113

22. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s7p7.pg

At what point does the normal to \( y = -5 - 4x + 3x^2 \) at \((1, -6)\) intersect the parabola a second time? (_______, ________)

The normal line is perpendicular to the tangent line. If two lines are perpendicular their slopes are negative reciprocals – i.e. if the slope of the first line is \( m \) then the slope of the second line is \(-1/m\).

Correct Answers:
- 0.166666666666667
- -5.58333333333333


The function \( f(x) = -2x^3 + 33x^2 - 60x + 8 \) has one local minimum and one local maximum. It is helpful to make a rough sketch of the graph to see what is happening.

This function has a local minimum at \( x = \) ___ with value \( f(x) = \) ________, and a local maximum at \( x = \) ___ with value \( f(x) = \) ________.

Correct Answers:
- 1
- -21
- 10
- 708


Consider the function \( f(x) = 3x^3 - 3x \) on the interval \([-2, 2]\).

Find the average or mean slope of the function on this interval.

By the Mean Value Theorem, we know there exists at least one \( c \) in the open interval \((-2, 2)\) such that \( f'(c) \) is equal to this mean slope.

For this problem, there are two values of \( c \) that work. The smaller one is ________, and the larger one is ________.

Correct Answers:
- -1.15470053837925
- 1.15470053837925


Consider the function \( f(x) = 2x^3 - 6x^2 - 90x + 8 \) on the interval \([-5, 10]\). Find the average or mean slope of the function on this interval.

By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-5, 10)\) such that \( f'(c) \) is equal to this mean slope. For this problem, there are two values of \( c \) that work. The smaller one is ________, and the larger one is ________.

Correct Answers:
- 30
- -3.58257569495584
- 5.58257569495584


Consider the function \( f(x) = 12x^3 + 75x^4 - 120x^3 + 3 \).

\( f(x) \) has inflection points at (reading from left to right) \( x = D, E, \) and \( F \)

where \( D \) is ________, and \( E \) is ________, and \( F \) is ________.

For each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).

\((-∞, D)\): ________
\([D, E]\): ________
\([E, F]\): ________
\([F, ∞)\): ________

Correct Answers:
- -4.42757223208277
- 0
- 0.677572232082767
- CD
- CU
- CD
- CU

27. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s7p14.pg

A right circular cone is to be inscribed in another right circular cone of given volume, with the same axis and with the vertex of the inner cone touching the base of the outer cone.

Draw a picture of the cones.

What must be the ratio of their altitudes for the inscribed cone to have maximum volume? ________

Correct Answers:
- 0.333333333333333
I have enough pure silver to coat 4 square meter of surface area. I plan to coat a sphere and a cube. Allowing for the possibility of all the silver going onto one of the solids, what dimensions should they be if the total volume of the silvered solids is to be a maximum?

The radius of the sphere is ______
and the length of the sides of the cube is ______.

Again, allowing for the possibility of all the silver going onto one of the solids, what dimensions should they be if the total volume of the silvered solids is to be a minimum?

The radius of the sphere is ______
and the length of the sides of the cube is ______.

Correct Answers:
• 0.564189583510922
• 0
• 0.330741786042061
• 0.661483572084122


One end of a ladder of length $L$ rests on the ground and the other end rests on the top of a wall of height $h$, as illustrated in the Figure on this page. As the bottom end is pushed along the ground towards the wall, the top end extends beyond the wall. The value of $x$ that maximizes the horizontal overhang $s$ is $x =$ _____________. (Your answer will depend on $L$ and $h$.)

In the particular case that $L = 18$ and $h = 7$ this value is $x =$ _____________.

The corresponding numerical value of $s =$ _____________.

Correct Answers:
• $(-h^*h+L^*h^*h)^{(2/3)}^{(1/2)}$
• 6.55514316683058

30. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210s7p17.pg
The illumination at a point is inversely proportional to the square of the distance of the point from the light source and directly proportional to the intensity of the light source. Suppose two light sources are $s$ feet apart and their intensities are $I$ and $J$, respectively. Let $P$ be the point between them where the sum of their illuminations is a minimum. The distance of $P$ from light source $I$ will be ____________ feet. (Your answer will depend on $I$, $J$, and $s$.)

Correct Answers:
• $s*I^{(1/3)}/(I^{(1/3)}+J^{(1/3)})$

A car traveling at 49 ft/sec decelerates at a constant 8 feet per second squared. How many feet does the car travel before coming to a complete stop?

Correct Answers:
• 150.0625

32. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210set8p4.pg
A ball is shot straight up into the air with initial velocity of 50 ft/sec. Assuming that the air resistance can be ignored, how high does it go? (Assume that the acceleration due to gravity is 32 ft per second squared.)

Correct Answers:
• 39.0625

Consider the function $f(x) = 6x^3 - 7x^2 + 5x - 1$.
An antiderivative of $f(x)$ is $F(x) = Ax^4 + Bx^3 + Cx^2 + Dx$ where $A$ is ____ and $B$ is ____ and $C$ is ____ and $D$ is ____

Correct Answers:
• 1.5
• -2.33333333333333
• 2.5
• -1

34. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210set8p7.pg
Consider the function $f(x) = 36x^3 - 18x^2 + 12x - 5$. Enter an antiderivative of $f(x)$

Correct Answers:
• $9*x^4-6*x^3+6*x^2-5*x$
Given that the graph of \( f(x) \) passes through the point \((10, 8)\) and that the slope of its tangent line at \((x, f(x))\) is \(6x + 6\), what is \( f(3) \)?
Correct Answers:
- \(-307\)

37. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210set8p11.pg
Consider the function \( f(x) = 4x^9 + 9x^5 - 6x^2 - 4 \). Enter an antiderivative of \( f(x) \).
Correct Answers:
- \(0.4x^{10} + 1.5x^6 - 2x^3 - 4x\)

Consider the function \( f(x) = 9x^2 - 7x^5 \). Let \( F(x) \) be the antiderivative of \( f(x) \) with \( F(1) = 0 \). Then \( F(4) \) equals
Correct Answers:
- \(5.0068359375\)

Consider the function \( f(x) = \frac{7}{x^2} - \frac{10}{x} \). Let \( F(x) \) be the antiderivative of \( f(x) \) with \( F(1) = 0 \). Then \( F(x) = \)
Correct Answers:
- \(-7x^{-1} + (1.66666666666667)x^{-6} + 5.33333333333333x\)

40. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210set8p15.pg
Suppose
\[
f(x) = \frac{1}{(x - 1)^2}
\]
and \( F \) is an antiderivative of \( f \) that satisfies \( F(0) = 1 \). Then \( F(x) = \)
Correct Answers:
- \(1/(1-x)\)

41. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210set8p17.pg
This problem revisits our familiar formulas for vertically moving objects.
Suppose we know that
\[
h''(t) = -g, \quad h'(0) = V, \quad \text{and} \quad h(0) = H.
\]
Then \( h(t) = \)
Correct Answers:
- \(-g/2*t^2 + V*t + H\)

42. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210set8p18.pg
This problem is a very simple example of a differential equation: an equation that relates a function to one or more of its derivatives. You can solve this problem by doing some educated guessing. ("educated" means "remember what we did in the past.")
Suppose \( f \) is the function that satisfies
\[
f'(x) = -f^2(x)
\]
for all \( x \) in its domain, and
\[
f(1) = 1.
\]
Then \( f(x) = \)
Correct Answers:
- \(1/x\)

43. (10 pts) 1250Library/set4_Graphing_and_Maximum-Minimum_Problems/1210set8p23.pg
The rectangle with the largest area that can be enclosed in a circle of radius \( r \) is of course a square. Its sides have length \( s = \) and its area is \( A = \).
Consider now the ellipse defined by
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\]
The rectangle with the largest area that can be inscribed in that ellipse has a horizontal side of length \( \) a vertical side of length \( \) and an area \( \).
Hint: You can solve the ellipse problem by cranking the handle and proceeding as we did for several similar geometric problems. Or you can look at the results for the circle and make an inspired guess.
Correct Answers:
- \(\sqrt{2}*r\)
- \(2*r*r\)
- \(\sqrt{2}*a\)
- \(\sqrt{2}*b\)
- \(2*a*b\)
This problem set deals mostly with sums and simple integrals. However, it starts out with an intriguing minimization problem.

1. (10 pts) set5/cone.pg
You are going to make tanks in the shape of a right circular cone. They will hold various volumes, but the amount of sheet metal required to build the tank will be as small as possible subject to the requirement of holding the required volume. These tanks will forever after be known all over the world as the famous Hsiang-Ping Huang tanks.

The quotient radius/height of your tanks equals ____________.

Correct Answers:
• 0.353553390593274

2. (10 pts) 1250Library/set5_The_Integral/1210set8p20.pg
Compute the sum
\[ \sum_{i=1}^{98} i = \quad \text{___________}. \]

Correct Answers:
• 4851

3. (10 pts) 1250Library/set5_The_Integral/1210set8p21.pg
Compute the sum
\[ \sum_{i=1}^{59} (2i - 1) = \quad \text{___________}. \]

Correct Answers:
• 3481

4. (10 pts) 1250Library/set5_The_Integral/1210set8p22.pg
Compute the sum
\[ \sum_{i=1}^{n} (2i - 1) = \quad \text{___________}. \]

Correct Answers:
• \( n^2 \)

5. (10 pts) 1250Library/set5_The_Integral/c4s4p1.pg
If \( f(x) = \int_0^t t^4 \, dt \)
then
\[ f'(x) = \quad \text{__________} \]
\[ f'(6) = \quad \text{__________} \]

Hint: You don’t need to compute \( f \) to answer this question.

Correct Answers:
• \( x^4 \)
• 1296

6. (10 pts) 1250Library/set5_The_Integral/c4s4p5.pg
The value of \( \int_0^4 (x + 1)^2 \, dx \) is ________________.

Correct Answers:
• 41.3333333333333

7. (10 pts) 1250Library/set5_The_Integral/c4s4p6.pg
The value of \( \int_1^2 \frac{1}{x^2} \, dx \) is ____________.

Correct Answers:
• 0.5

8. (10 pts) 1250Library/set5_The_Integral/q1.pg
Enter a formula for
\[ \sum_{i=m}^{n} i = \quad \text{__________} \]
where 0 < \( m < n \) are positive integers.

Correct Answers:
• \( (n^2 + n - m^2 + m) / 2 \)

9. (10 pts) 1250Library/set5_The_Integral/q2.pg
Enter a formula for
\[ (\sum_{i=1}^{n} i^2) - \sum_{i=1}^{n} i^3 = \quad \text{__________} \]
where \( n \) is a positive integer.

Correct Answers:
• 0

10. (10 pts) 1250Library/set5_The_Integral/s4_4_20.pg
The value of \( \int_2^4 (18x^2 - 8x + 4) \, dx \) is ________________.

Correct Answers:
• 1850

11. (10 pts) 1250Library/set5_The_Integral/sc5_2_24.pg
Evaluate the integral below by interpreting it in terms of areas.
In other words, draw a picture of the region the integral represents, and find the area using high school geometry.
\[ \int_4^7 \sqrt{16 - x^2} \, dx = \quad \text{__________} \]

Correct Answers:
• 25.132741232
12. (10 pts) 1250Library/set5_The_Integral/5.2.28.png
Evaluate the integral by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using high school geometry.

\[ \int_0^9 (9x - 5) \, dx = \] 

Correct Answers:
- 322.27777777778

13. (10 pts) 1250Library/set5_The_Integral/sc5.3.17.png
Evaluate the integral

\[ \int_3^2 \sin(t) \, dt = \] 

Correct Answers:
- -0.573845660053303

14. (10 pts) 1250Library/set5_The_Integral/sc5.4.19a.png
Find the derivative of

\[ g(x) = \int_5^3 u^5 \, du \]

BR \( g'(x) = \) 

HINT:

\[ \int_5^3 u^5 \, du = \int_5^3 u^5 \, du + \int_5^0 u^5 \, du \]

Correct Answers:
- \( 3 \times (3x+5) / (3x-3) - 5 \times (5x+5) / (5x-3) \)

15. (10 pts) 1250Library/set5_The_Integral/ur_in_0.12.png
Consider the function \( f(x) = \frac{x^2}{4} + 7 \).

In this problem you will calculate \( \int_0^a \left( \frac{x^2}{4} + 7 \right) \, dx \) by using the definition

\[ \int_a^b f(x) \, dx = \lim_{n \to \infty} \left[ \sum_{i=1}^{n} f(x_i) \Delta x \right] \]

The summation inside the brackets is \( R_n \) which is the Riemann sum where the sample points are chosen to be the right-hand endpoints of each sub-interval.

Calculate \( R_n \) for \( f(x) = \frac{x^2}{4} + 7 \) on the interval \([0, 4]\) and write your answer as a function of \( n \) without any summation signs.

\[ R_n = \]

\[ \lim_{n \to \infty} R_n = \]

Correct Answers:
- \( 7 \times 4 + 4 \times 3 \times (n+1) \times (2n+1) / (6 \times 4 \times n^2) \)
- \( 33.3333333333333 \)

16. (10 pts) 1250Library/set5_The_Integral/ur_in_3.11.png
The velocity function is \( v(t) = -t^2 + 3t - 2 \) for a particle moving along a line. Find the displacement and the distance traveled by the particle during the time interval \([-2, 6]\).

Displacement = 

Distance traveled = 

Correct Answers:
- \(-42.66666666667\)
- \(43\)
Math 1250-2 Fall 2008
Hsiang-Ping Huang.
WeBWorK assignment number 6.
due 10/21/2008 at 11:59pm MDT.
This set has more on integrals and derivatives. To stay in sync, and have home works of roughly consistent size trigonometric functions will start only on the next home work.

1. (10 pts) 1250Library/set6_The_Integral/1210s3p7.pg
For what value of the constant \(c\) is the function \(f\) continuous on \((-\infty, \infty)\) where
\[
f(x) = \begin{cases} c & \text{if } x = 0 \\ x \sin \frac{1}{x} & \text{otherwise} \end{cases}
\]

**Hint:** Ask what is \( \lim_{x \to 0} f(x) \)?

**Correct Answers:**
- 0

2. (10 pts) 1250Library/set6_The_Integral/1210s3p8.pg
For what value of the constant \(c\) is the function \(f\) continuous on \((-\infty, \infty)\) where
\[
f(b) = \begin{cases} cb + 7 & \text{if } b \in (-\infty, 8] \\ cb^2 - 7 & \text{if } b \in (8, \infty) \end{cases}
\]

**Correct Answers:**
- 0.25

3. (10 pts) 1250Library/set6_The_Integral/1210s3p9.pg
In this problem we consider three functions \(f\). Each of them is continuous at \(x = 0\), i.e.,

\[
\lim_{x \to 0} f(x) = f(0).
\]

In order to show by the \(\varepsilon/\delta\) definition that this is true one has to give a definition of \(\delta\) in terms of \(\varepsilon\) such that

\[
|x - 0| < \delta \implies |f(x) - f(0)| < \varepsilon.
\]

Match these choices of \(\delta\)
1. \(\delta = \varepsilon^2\)
2. \(\delta = \varepsilon\)
3. \(\delta = \sqrt{\varepsilon}\)

with the functions so that that choice of \(\delta\) establishes continuity of the function at \(x = 0\). You can use each choice only once. Enter the reference numbers of the given functions in the appropriate answer boxes.

\[
f(x) = x: __
\]
\[
f(x) = x^2: __
\]
\[
f(x) = \sqrt{x}: __
\]

**Correct Answers:**
- 2
- 3
- 1

4. (10 pts) 1250Library/set6_The_Integral/1210s3p10.pg
Given
\[
f''(x) = 5x + 3
\]
and \(f'(0) = -3\) and \(f(0) = -5\).

Find \(f''(x) = \) _____________

and find \(f(4) = \) _____________

**Correct Answers:**
- \(5/2*x^2 + 3*x + -3\)
- 60.3333333333333

5. (10 pts) 1250Library/set6_The_Integral/1210s3p11.pg
Suppose
\[
f(x) = x + 1
\]
and \(F\) is an antiderivative of \(f\) that satisfies

\[
F(0) = 1.
\]

Then
\[
F(x) = \) _____________.

**Correct Answers:**
- \(x^2/2+x+1\)

6. (10 pts) 1250Library/set6_The_Integral/1220s1p11.pg
**A word problem.** What number exceeds its square by the maximum amount?

Answer: _____________

**Correct Answers:**
- 0.5

7. (10 pts) 1250Library/set6_The_Integral/1220s1p12.pg
**Ups and Downs.** Consider the function \(f(x) = x^3 - 3x^2 + 2x\).

This function is increasing on the interval \((-\infty, A)\) where \(A=\) _____________.

It has an inflection point at \(x = \) ___________. Call that point \(B\). For each of the following intervals, tell whether \(f(x)\) is concave up (type in CU) or concave down (type in CD).

\((-\infty, B): \) ____________

\((B, \infty): \) ____________

**Correct Answers:**
- 0.422649730810374
- 1
- CD
- CU
8. (10 pts) 1250Library/set6_The_Integral/1220s1p16.pg

**Definite Integrals.** Evaluate

\[ \int_1^5 (5x^2 - 8x + 4) \, dx = \text{__________.} \]

**Correct Answers:**
- 126.66666666667

9. (10 pts) 1250Library/set6_The_Integral/1220s1p17.pg

**The Fundamental Theorem of Calculus.** Use the Fundamental Theorem of Calculus to find the derivative of

\[ f(x) = \int_2^x \left( \frac{1}{2} t^2 - 1 \right)^8 \, dt \]

**f'(x) = _______________**

**Correct Answers:**
- 2*(1/2*x^4-1)^8*x

10. (10 pts) 1250Library/set6_The_Integral/1220s1p18.pg

**Areas.** Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.

\[ y = 7x, \quad y = 6x^2 \]

**Correct Answers:**
- 1.58796296296296

11. (10 pts) 1250Library/set6_The_Integral/1220s1p22.pg

**More on Areas.** Farmer Jones, and his wife, Dr. Jones, both mathematicians, decide to build a fence in their field, to keep the sheep safe. Being mathematicians they decide that the fences are to be in the shape of the parabolas \( y = 4x^2 \) and \( y = x^2 + 6 \). What is the area of the enclosed region?

**Correct Answers:**
- 41.3137084989848

12. (10 pts) 1250Library/set6_The_Integral/1220s1p23.pg

**Yet More on Areas.** There is a line through the origin that divides the region bounded by the parabola \( y = 3x - 8x^2 \) and the x-axis into two regions with equal area. What is the slope of that line?

**Hint:** Draw a picture, write the area between the parabola and the line in terms of the slope of the line, and solve for the slope.

**Correct Answers:**
- 0.618898422047701

13. (10 pts) 1250Library/set6_The_Integral/1220s1p27.pg

**How Far is That Point?** Let \( P \) be the point \((p, q)\) and \( L \) the line \( y = mx + b \).

The distance of \( P \) from \( L \) is \( d = \text{_______________} \)

(Of course, your answer will be in terms of \( m, b, p \) and \( q \).)

**Correct Answers:**
- sqrt((q-m*p-b)**2/(1+m**2))

14. (10 pts) 1250Library/set6_The_Integral/1220s1p30.pg

**Shooting Arrows.** The ideas we are practicing in these exercises can be put to practical use. My son likes to shoot arrows. He was wondering at what speed the arrow leaves the bow when it is released. So we went to a large deserted beach at the Great Salt Lake and shot the bow at an angle of 45 degrees. The arrow hit the ground at a distance of 600 feet. All of this was measured very roughly, but lets work with these figures, and let’s ignore air resistance. For simplicity let’s also assume that the arrow is released at an initial height of 0 feet. As usual, assume that gravity causes an object to accelerate at 32 feet per second squared.

The arrow leaves the bow at a speed of ________ miles per hour.

**Note:** My son and I did worry about the effect of air resistance. The arrows were sticking out of the sand where they had hit the beach. So my son shot another arrow downwards at an angle of 45 degrees straight into the sand. It penetrated to about the same depth as the arrows that had flown the distance. So they must have hit at about the same speed. We concluded that ignoring air resistance in this case was reasonable.

**Hint:** The speed of the arrow is measured in the direction in which it flies. The significance of shooting at an angle of 45 degrees is that initially the horizontal and the vertical components of the speed are equal. There are 3600 seconds in an hour and 5280 feet in a mile.

**Correct Answers:**
- 94.475498594666

15. (10 pts) 1250Library/set6_The_Integral/1220s1p31.pg

**The Columbia Accident.** On January 16, 2003, 81.7 seconds into the ascent, a piece of foam, weighing about 1.7 pounds, and being roughly the size, shape and weight of a large loaf of bread, separated from the bipod ramp of the space shuttle Columbia’s external fuel tank and impacted the leading edge of the shuttle’s left wing. At that time the shuttle was traveling at approximately 1,568 mph (Mach 2.46) and was at an altitude of 66,000 feet. Based on film evidence, the foam traveled the 58-foot distance from the ramp to the wing in 0.16 seconds. Assuming a constant acceleration relative to the body of the space shuttle of ________ feet per second squared, the foam hit the wing at a speed of ________ miles per second, or ________ miles per hour. Unbeknownst to the astronauts and observers on earth, it struck the wing hard enough to cause the destruction of the Columbia two weeks later during re-entry on February 1, 2003.

**Note:** A much more involved calculation estimated the speed of the impact to be about 530 mph. You can find very detailed information about all aspects of the Columbia accident in this official report.
Remember that we expect your answer to be within one tenth of one percent of what it considers the true answer. Calculate your answers based on the given data and assumptions to at least four digits.

**Hint:** Figure everything relative to the moving space shuttle, i.e., the origin is traveling with the shuttle. Acceleration is the derivative of velocity, and velocity is the derivative of displacement. In this case, the acceleration is constant (by assumption), and the initial velocity and displacement are 0. After 0.16 seconds the displacement is 58 feet. There are 3,600 seconds in an hour, and 5280 feet in a mile. As in many real problems, the problem statement contains extraneous information. You have to decide what parts of the information are pertinent.

**Correct Answers:**
- 4531.25
- 125
- 494.318181818182

**16. (10 pts) 1250Library/set6_The_Integral/p12.pg**

**Areas**

Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.

\[ y = 4x, \quad y = 6x^2 \]

**Correct Answers:**
- 0.296296296296296

**17. (10 pts) 1250Library/set6_The_Integral/q3.pg**

You may think you have seen this problem before, and indeed you have. This time, however, you need to simplify your answer before entering it.

Enter a formula for

\[ (\sum_{i=1}^{n} i)^2 - \sum_{i=1}^{n} i^3 = \text{____________} \]

where \( n \) is a positive integer.

**Correct Answers:**
- 0

**18. (10 pts) 1250Library/set6_The_Integral/q4.pg**

This question summarizes a few simple facts about polynomials. Recall that a polynomial \( p \) can be written in the form

\[ p(x) = \sum_{i=0}^{n} a_i x^i \]

where we assume that \( a_0 \neq 0 \). The degree of a polynomial is the largest exponent present, and so the degree of \( p \) is \( n \). Fill in the blanks in the following questions:

- The degree of \( p(x) \) is _____.
- The degree of \( \int p(x) \, dx \) is _____.
- The degree of \( p^2(x) \) (i.e., the square of \( p \)) is _____.
- The \((n+1)\)th derivative of \( p \) is _____.

**Correct Answers:**
- \( n+1 \)
- \( n \)
- \( 2n \)
- 0

**19. (10 pts) 1250Library/set6_The_Integral/q5.pg**

Use the Fundamental Theorem of Calculus to carry out the following differentiation:

\[ \frac{d}{dx} \int_{1}^{\sqrt{x}} t \, dt = \text{____________} \]

**Correct Answers:**
- \( \sqrt{x} \cdot \frac{1}{2\sqrt{x}} \)

**20. (10 pts) 1250Library/set6_The_Integral/s4_4.21.pg**

The value of \( \int_{2}^{3} (4 - x^2) \, dx \) is ____________.

**Correct Answers:**
- 10.6666666666667

**21. (10 pts) 1250Library/set6_The_Integral/s4_4.27.pg**

The value of \( \int_{2}^{3} 6x^2 + 6 \, dx \) is ____________.

**Correct Answers:**
- 561.003092865686

**22. (10 pts) 1250Library/set6_The_Integral/sc5_2.3.pg**

Consider the integral

\[ \int_{1}^{7} (4x^3 + 3x + 6) \, dx \]

(a) Find the Riemann sum for this integral using right endpoints and \( n = 3 \).

(b) Find the Riemann sum for this same integral, using left endpoints and \( n = 3 \).

**Correct Answers:**
- 790
- 370

**23. (10 pts) 1250Library/set6_The_Integral/z1.pg**

In the next few problems we will explore the mathematics of motion with constant acceleration, and apply it to falling objects and accelerating and decelerating vehicles. Unless otherwise stated assume that we use feet and seconds as our units for length and time. Some of the problems may not require Calculus (i.e., differentiation or integration) for their solution, but that’s OK, you want to be able to solve all problems that come along.

As a warmup, let’s practice some conversions. A speed of \( m \) miles per hour equals a speed of ______ feet per second, and a speed \( f \) feet per second equals a speed of _______ miles per hour. Recall that there are 5280 feet in a mile, and 3600 seconds in an hour.

Recall that velocity is the derivative of distance, and acceleration is the derivative of velocity. Suppose that acceleration is constant, and equal to \( A \). Suppose at time 0 your velocity is \( V \).

(To accommodate WeBWorK idiosyncrasies, I’m using upper case to denote constants.)

Let \( v(t) \) denote your velocity at time \( t \). Then

\[ v(t) = \text{___________} \]
Suppose at time $t = 0$ your distance (in whatever direction you are moving) is $D$.

Let $d(t)$ denote your distance at time $t$. Then $d(t) = \ldots$.

Let’s translate that into a familiar context. Suppose $A = -32$ feet per second squared, $h(t)$ is your height at time $t$, $H$ is your initial height, and $V$ is your initial velocity. Then your height $h(t)$ at time $t$ is $h(t) = \ldots$, your velocity at time $t$ is $v(t) = h'(t) = \ldots$, and your acceleration at time $t$ is, of course, $a(t) = v'(t) = h''(t) = \ldots$ feet per second squared.

Correct Answers:
- $\cdot m*5280/3600$
- $\cdot f*3600/5280$
- $\cdot A*t + V$
- $\cdot A*t*t/2 + V*t +D$
- $\cdot -16*t*t + V*t +H$
- $\cdot -32*t + V$
- $\cdot -32$

24. (10 pts) 1250Library/set6_The_Integral/z2.pg

Suppose now you are driving along the highway at a speed $v$ feet per second. You apply the brakes which cause a constant deceleration $A$ feet per second squared. You come to a stop after $\ldots$ seconds. (Enter an expression that assumes a positive value for $A$.

A typical safe deceleration of a moving car is about one half $g$, half the acceleration of gravity, i.e., 16 feet per second squared.

Suppose you are driving at 60 miles per hour. You press on the brakes and decelerate at 16 feet per second squared. It will take you $\ldots$ seconds to stop. During that time you will travel $\ldots$ feet.

Assuming that it takes you 1 second to react to an emergency before you start braking, at the same initial speed, and the same constant deceleration, you will travel a total of $\ldots$ feet, before coming to a stop.

Correct Answers:
- $\cdot v/A$
- $\cdot 5.5$
- $\cdot 242$
- $\cdot 330$

25. (10 pts) 1250Library/set6_The_Integral/z3.pg

Let’s now replay that scenario at 80 miles per hour.

Again decelerating at 16 feet per second squared it will take you $\ldots$ seconds to stop.

During that time you will travel $\ldots$ feet.

Assuming that it takes you 1 second to react to an emergency before you start braking, at the same initial speed, and the same constant deceleration, you will travel a total of $\ldots$ feet, before coming to a stop.

Correct Answers:
- $\cdot 7.33333333333333$
- $\cdot 430.222222222222$
- $\cdot 547.555555555556$

26. (10 pts) 1250Library/set6_The_Integral/z4.pg

You are testing your brandnew Ferrari Testarossa. To see how well the brakes work you accelerate to 100 miles per hour, slam on the brakes, and determine that you brought the car to a stop over a distance of 481 feet. Assuming a constant deceleration you figure out that that deceleration is $\ldots$ feet per second squared. (Enter a positive number.)

I trust that you don’t have the courage to try this, but that night you wonder how long it would take you to stop (with the same constant deceleration) if your were moving at 200 miles per hour. Remembering your Calculus you figure out that your stopping distance would be $\ldots$ feet. (Enter a number, not an arithmetic expression.)

Correct Answers:
- $\cdot 22.3608223608224$
- $\cdot 1924$

27. (10 pts) 1250Library/set6_The_Integral/z5.pg

According to Richard Graham in "SR-71 Revealed: The Inside Story", the Lockheed SR-71 Reconnaissance plane accelerates from a standing start to a speed of Mach 3 (3 times the speed of sound) "in about 14 minutes". (During that time it also climbs to an altitude of about 70,000 feet, so the acceleration is not as impressive as you might otherwise expect.) Assuming that the time is exactly 14 minutes, and the speed of sound is 1100 feet per second, this is an average acceleration of $\ldots$ feet per second squared. Assuming that the acceleration is constant, during that time your SR-71 covers a distance of $\ldots$ miles.

By the way, you can see an SR-71 on display in the Hill Air Force Museum in Roy. It looks a little dusty, and it’s clear that its best days are over, but the plane is fiercely elegant and powerful. It still presents an impressive sight.

Correct Answers:
- $\cdot 3.92857142857143$
- $\cdot 262.5$
1. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p2.pg
Let
\[ f(x) = -7\cos x + 9\tan x. \]
Then
\[ f'(x) = \text{__________}. \]
Correct Answers:
- \((-1)*(-7*\sin x + 9/(\cos x)^2)\)

2. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p4.pg
If
\[ f(x) = 2\sin x \sqrt{1 + x^2} \]
then
\[ f'(x) = \text{__________}. \]
Correct Answers:
- \((2*\sin x * (4*\cos(x)*x + 4*\cos(x)*x + \sin(x)))/((\sqrt{x^2 + 1})*(x^2 + 1))\)

3. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p9.pg
If
\[ f(x) = \sin(x^2) \]
then
\[ f'(x) = \text{__________}. \]
Correct Answers:
- \((\cos(x^2))*2*x^2 \)

4. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p10.pg
If
\[ f(x) = \sin^3 x \]
then
\[ f'(x) = \text{__________} \]
and \[ f''(1) = \text{__________} \]
Correct Answers:
- \((3*\sin(x) * (3-1)) * \cos(x) \)
- \(1.14772110185144 \)

5. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p16.pg
If
\[ f(x) = \cos(\sin(x^2)), \]
then
\[ f'(x) = \text{__________}. \]
Correct Answers:
- \(-\sin(\sin(x^2))*\cos(x^2)*2*x \)

6. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p17.pg
The purpose of this problem is to show pretty much all of our rules at work at once.
If
\[ f(x) = \frac{x\sin^2x^2}{\sqrt{1+x^2}} \]
then \[ f'(x) = \text{__________} \]
Correct Answers:
- \((\sin(x^2)^2)*(4*\cos(x^2)*x^2 + 4*\cos(x^2)*x^2 + \sin(x^2)^2))/((\sqrt{x^2 + 1})*(x^2 + 1))\)

7. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p18.pg
Let
\[ f(x) = \tan x \]
Then
\[ f'(x) = \text{__________} \]
\[ f''(x) = \text{__________} \]
\[ f'''(x) = \text{__________} \]
Correct Answers:
- \(1/\cos(x)^2\)
- \(2*\sin(x)/\cos(x)^3\)
- \((2+4*\sin(x)^2)/\cos(x)^4\)
- \((16*\cos(x)^2*\sin(x)+24*\sin(x)^3)/\cos(x)^5\)

8. (10 pts) 1250Library/set7_Trigonometric_Functions/1210s4p21.pg
Directions (or bearings) on Earth are measured in degrees, running from zero to 360 degrees, clockwise, starting with 0 degrees being due North. So due East for example, is 90 degrees, due South 180 degrees, and Northwest is 315 degrees.

You are swinging a rock clockwise (looking from above) around your head and you are trying to hit a broom stick 15 feet due east of you. The rock moves in a circle of a radius of 3 feet around your head. When you release your sling the rock will continue to move along the tangent to the circle though its position at the time of the release. When you release the rock the sling is pointing in a direction of _____ degrees. Ignore the vertical motion of the rock.

It’s unrealistic, but remember that unless otherwise stated WeBWorK expects your answer to be within one tenth of one percent of the true answer.

**Hint:** You can solve this problem using calculus and computing the tangent to a circle. However, you can also solve it using plain trigonometry. The moral is that you want to use whatever requires the least amount of fuss for the problem at hand.

Correct Answers:
- \(11.5369590328155 \)
9. (10 pts) Let \( f(x) = \sin \frac{1}{x} \).

\[ f'(x) = \quad \]

Let \( g(x) = \frac{1}{\sin x} \).

\[ g'(x) = \quad \]

Correct Answers:
- \(-\cos(1/x)/(x^2)\)
- \(-\cos(x)/\sin(x)^2\)

10. (10 pts) Let \( f(x) = \tan^2 x \).

\[ f'(x) = \quad \]

Let \( g(x) = \tan^2 x \).

\[ g'(x) = \quad \]

Correct Answers:
- \(2x/\cos(x)^2\)
- \(2\tan(x)/\cos(x)/\cos(x)\)

11. (10 pts) Find \( y' \) by implicit differentiation. Match the expressions defining \( y \) implicitly with the letters labeling the expressions for \( y' \).

\[ \begin{align*}
\text{1. } \quad 3 \cos(x - y) &= 6y \cos x \\
\text{2. } \quad 3 \cos(x - y) &= 6y \sin x \\
\text{3. } \quad 3 \sin(x - y) &= 6y \sin x \\
\text{4. } \quad 3 \sin(x - y) &= 6y \cos x
\end{align*} \]

\[ \begin{align*}
\text{A. } \quad 3 \cos(x - y) + 6y \sin x &= 3 \cos(x) + 6y \cos x \\
\text{B. } \quad 3 \cos(x - y) - 6y \cos x &= 3 \cos(x - y) + 6y \sin x \\
\text{C. } \quad -3 \sin(x - y) + 6y \sin x &= 6 \cos x - 3 \sin(x - y) \\
\text{D. } \quad -3 \sin(x - y) - 6y \cos x &= 6 \sin x - 3 \sin(x - y)
\end{align*} \]

Correct Answers:
- C
- D
- B
- A

12. (10 pts) A plane flying with a constant speed of 27 km/min passes over a ground radar station at an altitude of 10 km and climbs at an angle of 45 degrees. At what rate, in km/min is the distance from the plane to the radar station increasing 3 minutes later? Your answer: \( \quad \) kilometers per minute.

**Hint:** The law of cosines for a triangle is

\[ c^2 = a^2 + b^2 - 2ab \cos(\theta) \]

where \( \theta \) is the angle between the sides of length \( a \) and \( b \).

Correct Answers:
- 26.9133948328552

13. (10 pts) Suppose \( f(x) = x^2 + \sin x \)

and \( F \) is an antiderivative of \( f \) that satisfies

\[ F(\pi) = 0. \]

Then \( F(x) = \quad \).

Correct Answers:
- \( x^{3/3} - 3\cos(x) - 1 - 3.14159265358979^3/3 \)

14. (10 pts) Suppose \( f \) is the function that satisfies

\[ f''(x) = -f(x) \]

for all \( x \) in its domain, \( f(0) = 0 \), and \( f'(0) = 1 \). Then

\[ f(x) = \quad \]

Correct Answers:
- \( \sin(x) \)

15. (10 pts) The Fundamental Theorem of Calculus. Use the Fundamental Theorem of Calculus to find the derivative of

\[ f(x) = \int_{5}^{x} \left( \frac{1}{5^2} - 1 \right)^{11} dr \]

\[ f'(x) = \quad \]

Correct Answers:
- \( 2*(1/5*x^4-1)^{11}*x \)
Find a formula for $f^{-1}(x)$ if:

- $f(x) = -\frac{x}{3} + 1$
- $f(x) = -\sqrt{1-x}$
- $f(x) = \sqrt{\frac{1}{1-x^2}}$
- $f(x) = \frac{1}{x^{**2} + 2}$
- $f(x) = \frac{1}{x^{**3}}$

**Hint:** Undo the operations of $f(x)$ from the outside in. Undo the cubed operation first.

**Correct Answers:**
- $\frac{1}{x^{**}(1/3) + 1} / (x^{**}(1/3) - 3)$
- $2/(3(x^{**}(1/3)-1)*)2*x^{**}(2/3))$

**20. (10 pts) 1250Library/set7_Trigonometric_Functions/1220s5p5.pg**

Applying your knowledge of Inverse Functions. A ball is thrown vertically upward from the ground with velocity $s$. Find the maximum height $H$ of the ball as a function of $s$. Then find the velocity $s$ required to achieve a height of $H$. As usual, ignore air resistance, and assume that gravity causes a downward acceleration of 32 feet per second squared.

$H =$ ____________.

$s =$ ____________.

**Hint:** The velocity of the ball is $v(t)$ a function of time, $t$. It is given by $v(t) = s - 32t$. Integrate to get a function describing the height of the ball as a function of time. This also determines the height of the ball as a function of $s$. Use techniques you remember from Calculus I to maximize this function. For the second part, let $H$ be some given height and set this equal to $H(s)$. Then use techniques of this chapter to find a value of $s$ which achieves the height $H$. Remember that Mathematics and WeBWorK are case sensitive. So you need to use an upper case $H$ and a lower case $s$.

**Correct Answers:**
- $(s**2)/64$
- $8*H^{**}.5$

**21. (10 pts) 1250Library/set7_Trigonometric_Functions/1220s5p4.png**

Evaluate $\int \tan(x)dx$.

Answer: ________

**Hint:** Recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and use logarithms.

**Correct Answers:**
- $0.693147180559945$

**22. (10 pts) 1250Library/set7_Trigonometric_Functions/7p23.png**

This entertaining problem does not require Calculus for its solution. It is included here to help set the stage for our next topic: the Calculus of exponential and logarithmic functions.

Your village depends for its food on a nearby fishpond. One day the wind blows a water lily seed into the pond, and the lily begins to grow. It doubles its size every day, and its growth is such that it will cover the entire pond (and smother the fish population) in 45 days. You are away on vacation, and the villagers depend on your vigilance and intelligence to handle their affairs. Without you present they will take action about that lily only on regular work days, and only when the lily covers half the pond or more. The lily will cover half the pond ______ days after it first starts growing. The day it reaches that size happens to be a holiday....
Hint: You don’t need algebra for this problem, just common sense.
Correct Answers:
• 44

23. (10 pts) 1250Library/set7_Trigonometric_Functions/sc5_4_18.pg
Find the derivative

\[ h'(x) = \text{________________________} \]

of

\[ h(x) = \int_{-1}^{\sin(x)} (\cos(t^3) + t) \, dt \]

Correct Answers:
• \( \cos(x) \cdot (\cos(\{(\sin(x))^3\}) + \sin(x)) \)
Math 1250-2 Fall 2008

Hsiang-Ping Huang.

WeBWorK assignment number 8.

due 11/04/2008 at 11:59pm MST.

This set is on exponentials and logarithms.

Note that in WeBWorK \( \ln(x) \) and \( \log(x) \) both mean the natural logarithm. As we discussed in class, to compute the logarithm of \( x \) to any base \( a \) use the formula
\[
\log_a(x) = \frac{\ln(x)}{\ln(a)}.
\]

You can enter powers like \( a^x \) as \( a**x \). WeBWorK does know the base \( e \) of the natural exponential, and you can also enter the natural exponential as \( \exp(x) \).

1. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p1.pg

(Section 7.1, No 4). Find the indicated derivative.

\[ D_x \ln(3x^2 + 2x) = \]

**Hint:** Recall the definition of the derivative of the natural logarithm function.

**Correct Answers:**

\[ \frac{(9x^2+2+2x)}{(3x^1 + 2x)} \]

2. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p2.pg

(Section 7.1, No 6). Find the indicated derivative.

\[ D_x \ln(\sqrt{3x-2}) = \]

**Hint:** This is similar to the previous problem. Use the chain Rule.

**Correct Answers:**

\[ \frac{3}{6x-4} \]

3. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p3.pg

Section 7.1, No 8. Understand basic derivatives. Find \( y' \) if

\[ y = x^2 \ln x \]

**Hint:** Use the Product and Chain Rules.

**Correct Answers:**

\[ 2x^1 \ln(x) + x \]

4. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p4.pg

Section 7.1, No 10. This is a more complicated differentiation exercise which will involve all three of our main rules: the product, quotient, and chain rules. You’ll find the right derivative by proceeding carefully and deliberately. You may want to edit the resulting expression in a separate file and then cut and paste it into the answer window. Find

\[
\frac{dr}{dx}
\]

if

\[
r = \frac{\ln x}{x^2 \ln x^2} + (\ln \frac{1}{x})^3
\]

**Correct Answers:**

\[ \frac{(x^1 \ln(x^2)) - (2x^3 \ln (x^2) + 2x^3) + \ln (x))}{x^4 / (\ln (x^2))^2 - 3*(\ln (\frac{1}{x}))^2} / x \]

5. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p5.pg

Section 7.1, No 16. Logarithms as anti-derivatives. As you know, indefinite integrals have an undetermined constant. In these exercises, use any numerical integration constant you like, such as 0. Do not use a symbol, such as C.

\[
\int \frac{1}{1-2x} dx = \]

**Hint:** Think about the natural log function.

**Correct Answers:**

\[ -.5*\ln(1-2x) \]

6. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p6.pg

Section 7.1, No 18. Logarithms as anti-derivatives. Use any numerical integration constant you like, such as 0. Do not use a symbol, such as C.

\[
\int \frac{z}{2z^2 + 8} dz = \]

**Hint:** Use the natural log function, and notice that the numerator is closely related to the derivative of the denominator.

**Correct Answers:**

\[ .25*\ln(2z^2+8) \]

7. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p7.pg

Section 7.1, No 20. Logarithms as anti-derivatives.

\[
\int \frac{-1}{x(lnx)^2} dx = \]

**Hint:** Use the natural log function and substitution.

**Correct Answers:**

\[ 1/1\ln(x) \]
Section 7.1, No 38 A Word Problem. The rate of transmission in a telegraph cable is observed to be proportional to

\[ x^2 \ln\left(\frac{1}{x}\right) \]

where \( x \) is the ratio of the radius of the core to the thickness of the insulation \((0 < x < 1)\). What value of \( x \) gives the maximum rate of transmission?

**Correct Answers:**

\[ \frac{1}{2.718281828}^{0.5} \]

**Hint:** Think about what does 'proportional' means in this case. Essentially, this means that the rate of transmission is a constant multiple of the expression given. Write the rate, \( R \), this way, differentiate and apply your knowledge of rates of change and maxima and minima to solve for \( x \).

---

9. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s2p10.pg

Suppose

\[ f(x) = x^x \]

In expressions like this one, conventionally the upper power is computed first, i.e.,

\[ f(x) = x(x^x) \]

Then

\[ f'(x) = \]

**Hint:** Recall how we differentiated \( x^x \) in class.

**Correct Answers:**

\[ x \cdot x^x(x \cdot x) - (1/x + \log(x) \cdot 2 + \log(x)) \]

10. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s4p1.pg

(Section 7.3, No 12). Find \( D_x y \) if \( y = e^{2x-x} \)

**Correct Answers:**

\[ (4x - 1) \cdot e^{x \cdot 2 - x} \]

11. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s4p2.pg

(Section 7.3, No 14). Find \( D_x y \) if

\[ y = e^{-1/x^2} \]

**Correct Answers:**

\[ -5 \]

**Hint:** Recall the definition of the derivative of the natural exponential function and use the chain rule.

**Correct Answers:**

\[ (2/x^3) \cdot e^{x \cdot (-1/x^2)} \]

12. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s4p3.pg

(Section 7.3, No 20). Find \( D_y y \) if \( y = e^{1/x^2} + \frac{1}{e^{x^2}} \)

**Correct Answers:**

\[ (-2/x^2) \cdot e^{1/(x^2)} + (-2x) / (e^{x^2}) \]

**Hint:** Apply the chain rule and watch for the right sign.

**Correct Answers:**

\[ \frac{5}{e^{x^2} - 3} \]

13. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s4p4.pg

(Section 7.3, No 30). Compute the integral. You can use any integration constant you like, including 0. Do not use a variable like \( C \).

\[ \int xe^{x^2-3}d x = \]

**Hint:** Use substitution and recall the example in the textbook of how to integrate the exponential function.

**Correct Answers:**

\[ \ln(e^{x^2} - 1) \]

14. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s4p5.pg

(Section 7.3, No 32). Compute the integral

\[ \int e^{x} - 1 dx = \]

**Hint:** Use substitution and remember what you know about the natural logarithm.

**Correct Answers:**

\[ \ln(e^{x^2} - 1) \]

15. (10 pts) 1250Library/set8_Exponentials_and_Logarithms-/1220s5p1.pg

You should be able to do the first two problems without using a calculator. Use the basic definitions of logarithms and exponentials. If you see a base 3 logarithm, for example, compute the first few powers of 3, and see how they relate to the number of which you are taking the logarithm.

Evaluate the following expressions.

(a) \( \log_3 \left( \frac{1}{3} \right) = \)

(b) \( \log_4 1 = \)

(c) \( \log_4 \sqrt{64} = \)

(d) \( 8^{\log_8 14} = \)

**Correct Answers:**

\[ -5 \]

\[ 0 \]

\[ 1.5 \]

\[ 14 \]
16. (10 pts) Evaluate the following expressions.
   (a) \( \ln e^9 = \) ____
   (b) \( e^{\ln 2} = \) ____
   (c) \( e^{\ln \sqrt{3}} = \) ____
   (d) \( \ln(1/e^5) = \) ____
   Correct Answers:
   - 9
   - 2
   - 1.73205080756888
   - -5

17. (10 pts) Suppose \( y = e^{1/x^2} + 1/e^{x^2} \). Find \( D_x y \).
   \( D_x y = \) __________________________
   Hint: Apply the chain rule and watch for the right sign.
   Correct Answers:
   - \(-2\exp(1/(x^{*2}))/((x^{*3}) - 2x/\exp(x^{*2})

18. (10 pts) Human hair from a grave in Africa proved to have only 66\% of the carbon 14 of living tissue. When was the body buried? The half life of carbon 14 is 5730 years. See Problem 13 of Section 7.5 of the course text. The body was buried about ________ years ago.
   Hint: From the text we learn the half-life of Carbon-14 is 5730 years. Use this and the the information about 66\% to help you find \( r \) as in exercise 1 of this same set. Then find \( t \).
   Correct Answers:
   - 3434.9176348523

19. (10 pts) Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at 308\degree F and left to cool in a room at 74\degree F, its temperature \( T \) after \( t \) hours will satisfy the differential equation
   \[
   \frac{dT}{dt} = k(T - 74).
   \]
   If the temperature fell to 209\degree F in 0.9 hour(s), what will it be after 3 hour(s)?
   After 3 hour(s), the temperature will be _____ degree F.
   Hint: Separate variables. Work through example 2, p. 342 as it is a similar problem.
   Correct Answers:
   - 111.406082527677
23. (10 pts) Assume the world population will continue to grow exponentially with a growth constant \( k = 0.0132 \) (corresponding to a doubling time of about 52 years), it takes \( \frac{1}{2} \) acre of land to supply food for one person, and there are 13,500,000 square miles of arable land in the world.

How long will it be before the world reaches the maximum population? Note: There were 6.06 billion people in the year 2000 and 1 square mile is 640 acres.

Answer: The maximum population will be reached some time in the year ______.

Hint: Convert .5 acres of land per person (for food) to the number of square miles needed per person. Use this and the number of arable square miles to get the maximum number of people which could exist on Earth. Proceed as you have in previous problems involving exponential growth.

Correct Answers:
- 2079

24. (10 pts) Let \( f(x) = x^2 \tan^{-1}(2x) \)

\[ f'(x) = \] 

NOTE: The WeBWorK system will accept \( \arctan(x) \) but not \( \tan^{-1}(x) \) as the inverse of \( \tan(x) \).

Hint: use the product and chain rules and recall the derivative of \( \tan^{-1}(x) \).

Correct Answers:
- \( 2x^{*x^2}(2-1)*\arctan(2*x) + x^{*x^2}/(1+2*x^2*x^2) \)

25. (10 pts) Let \( f(x) = 5 \sin(x) \sin^{-1}(x) \)

\[ f'(x) = \] 

NOTE: The WeBWorK system will accept \( \arcsin(x) \) and not \( \sin^{-1}(x) \) as the inverse of \( \sin(x) \).

Hint: Use the product rule and recall the derivative of \( \sin^{-1}(x) \).

Correct Answers:
- \( 5*(\cos(x)*\sin(x)+\sin(x)/sqrt(1-x^2)) \)

26. (10 pts) Evaluate the integral:

\[ \int \frac{dx}{\sqrt{3x\sqrt{x^2-1}}} = \] 

Hint: Recall the derivative of \( \sec^{-1}(x) \).

Correct Answers:
- \( \arctan(e^{*x}) \)
Section 7.7 No 76: An airplane in Australia is flying at a constant altitude of 2 miles and a constant speed of 600 miles per hour on a straight course that will take it directly over a kangaroo on the ground. How fast is the angle of elevation of the kangaroo’s line of sight increasing when the distance from the kangaroo to the plane is 3 miles? Give your answer in radians per minute.

**Answer:** __________

**Hint:** The angle of elevation is that angle that the line of sight makes with the horizontal. Construct a triangle representing the situation. Use this to get an equation first involving \( x \) and \( y \), then differentiate to involve \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \). Think about what it means that the plane is getting closer to the kangaroo.

**Correct Answers:**
- 2.22222222222222

32. (10 pts) 1250Library/set8_Exponentials_and_Logarithms/intro.pg

This first problem is just an exercise in entering logarithmic and exponential functions into WeBWorK. Remember that the only logarithm that WeBWorK knows is the natural logarithm, and you write it as \( \log(x) \). Any other logarithm needs to be expressed in terms of the natural logarithm. You can use a double asterisk or a caret \( ^\ast \) for exponentiation. The number \( e \) in WeBWorK is \( e \) and the natural exponential can also be written as \( \exp(x) \).

Try these notations by entering the following functions:

\[
\begin{align*}
    f(x) &= e^x \\
    f(x) &= \ln(x) \\
    f(x) &= 2^x \\
    f(x) &= b^x \\
    f(x) &= \log_2(x) \\
    f(x) &= \log_b(x)
\end{align*}
\]

**Correct Answers:**
- \( \exp(x) \)
- \( \log(x) \)
- 2\(^x\)
- \( b\(^x\)\)
- \( \log(x)/\log(2) \)
- \( \log(x)/\log(b) \)

33. (10 pts) 1250Library/set8_Exponentials_and_Logarithms/rope.pg

A free hanging cylindrical rope will break under its own weight if it exceeds a certain critical length. Suppose you have a well that’s deeper than the critical length of your rope, and you need a rope that will reach the bottom of that well.

A rope will break at any point if the stress at that point exceeds a certain critical value. The stress is the ratio of the weight below that point and the area of the cross section at that point. Thus it does not help to increase the radius of the rope by a certain factor. You’d increase the weight and the area of the cross section by the square of that factor. When computing the stress those squares would cancel. The critical length of a cylindrical rope is independent of its radius.

However, you can increase the depth a rope can reach by increasing its radius towards the top. This problem explores that idea. The rope will break when the stress \( s \) at a certain point exceeds a specific value \( c \). That critical value \( c \) depends on the material of which the rope is made. So we want to construct a rope which has a constant stress everywhere along the rope, and that carries a weight \( w \), say.

Putting this information into mathematical terms we obtain the equation

\[
\int_0^x u \pi r^2(t) \, dt + w = c
\]

or

\[
u \int_0^x \pi r^2(t) \, dt + w = c \pi r^2(x)
\]

for all \( x \). Here

\( u \) is the specific weight of the rope
\( r(x) \) is the radius of the rope at a distance \( x \) measured upwards from its bottom
\( w \) is the weight carried by the rope.
\( c \) is the critical stress (incorporating any safety factors).

Differentiate with respect to \( x \) in the above equation, obtain a differential equation for \( r \), determine \( r(0) \) by the fact that \( w \) is attached at \( x = 0 \) and enter the radius of your special rope:

\[
r(x) = ___________________________
\]

Of course, your answer will depend on \( x \), \( w \), \( u \), and \( c \).

**Correct Answers:**
- \((w/c/3.14159265358979)^{1/2} \cdot \exp(u/c/2^x)\)
In a well known story the inventor of the game of chess was asked by his well pleased King what reward he desired. ”Oh, not much, your majesty”, the inventor responded, ”just place a grain of rice on the first square of the board, 2 on the next, 4 on the next, and so on, twice as many on each square as on the preceding one. I will give this rice to the poor.” (For the uninitiated, a chess board has 64 squares.) The king thought this a modest request indeed and ordered the rice to be delivered.

Let \( f(n) \) denote the number of rice grains placed on the first \( n \) squares of the board. So clearly, \( f(1) = 1 \), \( f(2) = 1 + 2 = 3 \), \( f(3) = 1 + 2 + 4 = 7 \), and so on. How does it go on? Compute the next two values of \( f(n) \): \( f(4) = \) \( f(5) = \). Ponder the structure of this summation and then enter an algebraic expression that defines \( f(n) = \) as a function of \( n \).

Supposing that there are 25,000 grains of rice in a pound, 2000 pounds in a ton, and 6 billion people on earth, the inventor’s reward would work out to approximately \( \underline{61.4891469123652} \) tons of rice for every person on the planet. Clearly, all the rice in the kingdom would not be enough to begin to fill that request. The story has a sad ending: feeling duped, the king caused the inventor of chess to be beheaded.

**Hint:** Note the relationship between the number of grains on each square, and the number of grains on the preceding squares combined.

**Correct Answers:**
- 15
- 31
- \( 2^n - 1 \)
- 61.4891469123652
WeBWorK assignment number 9.
due 11/11/2008 at 11:59pm MST.

This set just has a few differentiation and integration exercises.

1. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s9p1.pg
Suppose \( y = \sinh(x^2 + x) \). Find \( D_x y \).
Answer: \( D_x y = \). 

Hint: Recall the derivative of \( \sinh(x) \) and use the chain rule.
Correct Answers:
• \((2x+1)\cosh(x^2+x)\)

2. (1 pt) set9/1220s9p2.pg
Suppose \( y = \ln(\coth x) \). Find \( D_x y \).
Answer: \( D_x y = \).

Hint: Compute the derivative of \( \cosh(x) = \frac{\cosh(x)}{\sinh(x)} \) and don’t ever forget the derivative of \( \ln u \)
Correct Answers:
• \( -\frac{\sech(x)}{\sinh(x)} \)

3. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s9p3.pg
Evaluate the integral
\[
\int \frac{\cosh \sqrt{z}}{\sqrt{z}} \, dz
\]
Answer: ________________.

Hint: Use Substitution.
Correct Answers:
• \( 2\sinh(\sqrt{z}) \)

4. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s10p1.pg
Perform the indicated integration.
\[
\int \frac{e^t}{2+e^t} \, dt = 
\]
Hint: Use substitution with \( u = 2 + e^t \)
Correct Answers:
• \( \ln(2 + e^{-x}) \)

5. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s10p2.pg
Perform the indicated integrations.
\[
\begin{align*}
\int \frac{e^t}{e^t+1} \, dx &= \\
\int \frac{e^t}{e^t+1} \, dx &= \\
\int \frac{e^t+1}{e^t} \, dx &= \\
\end{align*}
\]
Correct Answers:
• \( \ln(1 + e^{-x}) \)
• \( x/\exp(1) \)
• \( \exp(1) \log(e^{x}+1) \)

6. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s10p3.pg
\[
\int \frac{2^t}{\sqrt{1-t^4}} \, dt = 
\]
Hint: Use substitution with \( u = 1 - t^4 \)
Correct Answers:
• \( -(1-x**4)**.5 \)

7. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s10p4.pg
\[
\int e^{\cos z} \sin z \, dz = 
\]
Hint: Use substitution with \( u = \cos z \)
Correct Answers:
• \( -e^u \cos(z) \)

8. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s10p5.pg
\[
\int \frac{\sin(4t-1)}{1 - \sin^2(4t-1)} \, dt = 
\]
Hint: In the denominator, express the \( \sin \) in terms of the \( \cos \), and use substitution.
Correct Answers:
• \( \sec(4t-1)/4 \)

9. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s10p6.pg
\[
\int_0^1 \frac{e^{2t} - e^{-2t}}{e^{2t} + e^{-2t}} \, dx = 
\]
Hint: Try substitution with \( u = e^{2t} + e^{-2t} \)
Correct Answers:
• \( 0.662501373678932 \)

10. (1 pt) 1250Library/set9_Basic_Methods_of_Integration/1220s11p1.pg
\[
\int \sin^2(3x) \, dx = 
\]
Correct Answers:
• \( -1/6 \cos(3 \, x) \sin(3 \, x) + x/2 \)
11. \(\int x\cos^3(x^2)dx = \) ________________.
Correct Answers:
- \(\sin(x^2)*(-\sin(x^2)^2 + 3)/6\)

12. \(\int \ln(x^2)dx = \) ________________.
Correct Answers:
- \(2*(x*\ln(x)-x)\)

13. Use integration by parts to find the following:
\(\int xe^{3x}dx = \) ________________.
Hint: Use the substitution
\[u = x \quad \text{and} \quad dv = e^{3x}dx\]
Correct Answers:
- \(e^{3x} * (x/3 - 1/9)\)

14. Use integration by parts to find the following:
\(\int (t+7)e^{2t+3}dt = \) ________________.
Hint: Use the substitution
\[u = t+7 \quad \text{and} \quad dv = e^{2t+3}dt\]
Correct Answers:
- \(e^{2t+3} * ((t+7)/2 - 1/4)\)

15. Use integration by parts to find the following:
\(\int x\sin(2x)dx = \) ________________.
Correct Answers:
- \(-x/2*cos(2x) + sin(2x)/4\)

16. Use integration by parts to find the following:
\(\int e^a\sin(x)dx = \) ________________.
Hint: Use Integration by Parts twice.
Correct Answers:
- \((-\exp(a*x) \cos(x) + a*\exp(a*x) \sin(x))/(1+a^2) + 1/(1+a^2)\)

17. Evaluate the integral below. Your answer will of course involve a and b (and x). Don’t worry about the integration constant.
\(\int e^{ax}\cos(bx)dx = \) ________________
Correct Answers:
- \(\exp(a*x)*\left(\cos(b*x) + a*\sin(b*x)\right)/(a^2+b^2)\)

18. Find the following indefinite integrals:
(a) \(\int x(\sqrt{x}-1)dx = \) ________________ + C.
(b) \(\int x^3(\sqrt{x}-1)dx = \) ________________ + C.
(c) \(\int x^3(1/\sqrt{x}-1)dx = \) ________________ + C.
Correct Answers:
- \(3/7*(x-1)^{(7/3)} + 3/4*(x-1)^{(4/3)}\)
- \(2/9*(x-1)^{(9/2)} + 6/7*(x-1)^{(7/2)}\)
- \(2/7*(x-1)^{(7/2)} + 6/5*(x-1)^{(5/2)}\)
- \(2*(x-1)^{(3/2)} + 2*(x-1)^{(1/2)}\)

19. Evaluate the following definite integrals:
(a) \(\int_0^1 x^2e^xdx = \) ________________
(b) \(\int_0^1 x^2e^xdx = \) ________________
(c) \(\int_0^1 (\ln(x))^2dx = \) ________________
(d) \(\int_0^3 \cos^6(x)dx = \) ________________
Correct Answers:
- \(0.718\)
- \(0.565475572069307\)
- \(0.188317305596622\)
- \(0.4908734375\)
In this set you are asked to solve a few differential equations.

1. (10 pts) 1250Library/set10_Differential_Equations/1220s7p1.pg
Solve the following differential equation. As you know, indefinite integrals are used to solve these equations and have an undetermined constant. In this exercise use \( C = 0 \).

\[
\frac{dy}{dx} + 2y = x
\]

Use the formula:

\[
\int x e^{2x} \, dx = e^{2x} \left( \frac{x}{2} - \frac{1}{4} \right).
\]

Of course, you can check that formula easily by differentiation.

(Also, check your solution of the differential equation by differentiation.)

**Hint:** Recognize this as a first-order linear differential equation and follow the general method for solving these.

**Correct Answers:**
- \( .5x - .25 \)

2. (10 pts) 1250Library/set10_Differential_Equations/1220s7p2.pg
Solve the following differential equation. \( \frac{dy}{dx} = e^{2x} - 3y \) and \( y = 1 \) when \( x = 0 \)

\( y = \) ________

**Hint:** Recognize this as a first-order linear differential equation and follow the general method for solving these and use the initial conditions to find the integration constant.

**Correct Answers:**
- \( .2e^{x/4} + .2e^{-3x} \)

3. (10 pts) 1250Library/set10_Differential_Equations/1220s7p3.pg
A tank initially contains 200 gallons of brine, with 50 pounds of salt in solution. Brine containing 2 pounds of salt per gallon is entering the tank at the rate of 4 gallons per minute and is is flowing out at the same rate. If the mixture in the tank is kept uniform by constant stirring, find the amount of salt in the tank at the end of 40 minutes.

The amount of salt in the tank at the end of 40 minutes is ________ pounds.

**Correct Answers:**
- \( 242.734862537726 \)

4. (10 pts) 1250Library/set10_Differential_Equations/1220s7p4.pg
A tank initially contains 50 gallons of brine, with 30 pounds of salt in solution. Water runs into the tank at 6 gallons per minute and the well-stirred solution runs out at 5 gallons per minute. How long will it be until there are 25 pounds of salt in the tank?

The amount of time until 25 pounds of salt remain in the tank is ________ minutes.

**Hint:** This problem is a bit complicated. Set up a differential equation and separate variables.

**Correct Answers:**
- \( 1.8568464683241 \)

5. (10 pts) 1250Library/set10_Differential_Equations/q0.pg
Here are some initial value problems with obvious solutions, as discussed in class. In all cases the solutions are functions of \( x \).

All letters other than \( y \) and \( x \) denote constants.

The solution of \( y' = ky \), \( y(0) = A \) is \( y(x) = \) ________.

The solution of \( y'' = k^2y \), \( y(1) = y(-1) = A \) is \( y(x) = \) ________.

The solution of \( y'' = k^2y \), \( y(1) = -y(-1) = A \) is \( y(x) = \) ________.

The solution of \( y'' = -k^2y \), \( y(0) = 1 \), \( y'(0) = 0 \) is \( y(x) = \) ________.

The solution of \( y'' = -k^2y \), \( y(0) = 0 \), \( y'(0) = 1 \) is \( y(x) = \) ________.

The solution of \( y'' = -k^2y \), \( y(0) = A \), \( y'(0) = B \) is \( y(x) = \) ________.

**Correct Answers:**
- \( A \exp(kx) \)
- \( A \exp(kx) + B \exp(-kx) \)
- \( A \cos(kx) + B \sin(kx) \)
- \( \sin(kx)/k \)
- \( A \cos(kx) + B \sin(kx)/k \)
6. (10 pts) Library/set10_Differential_Equations/q1.pg
This is a warmup for the next question.
The solution of the differential equation
\[ y' = y(1-y), \quad y(0) = \frac{1}{2} \]
is
\[ y(t) = \quad \]
Hint: Separate variables. Note that
\[ \frac{1}{y(1-y)} = \frac{1}{y} + \frac{1}{1-y}. \]
Correct Answers:
• \( \exp(t)/(1+\exp(t)) \)

7. (10 pts) Library/set10_Differential_Equations/q2.pg
Consider the growth of a population \( p(t) \). It starts out with \( p(0) = A \). Suppose the growth is unchecked, and hence
\[ p' = kp \]
for some constant \( k \).
Then \( p(t) = \quad \)
Of course populations don’t grow forever. Let’s say there is a stable population size \( Q \) that \( p(t) \) approaches as time passes. Thus the speed at which the population is growing will approach zero as the population size approaches \( Q \). One way to model this is via the differential equation
\[ p' = kp(Q-p), \quad p(0) = A. \]
The solution of this initial value problem is \( p(t) = \quad \)
Hint: Proceed as in the last problem.
Correct Answers:
• \( A\exp(k*t) \)
• \( Q/A/(Q-A)*\exp(k*Q*t)/(1+A/(Q-A)*\exp(k*Q*t)) \)

8. (10 pts) set10/ur_de_2.7.pg
Suppose you have just poured a cup of freshly brewed coffee with temperature 90°C in a room where the temperature is 25°C. Newton’s Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Therefore, the temperature of the coffee, \( T(t) \), satisfies the differential equation
\[ \frac{dT}{dt} = k(T - T_{room}) \]
where \( T_{room} = 25 \) is the room temperature, and \( k \) is some constant.
Suppose it is known that the coffee cools at a rate of 2°C per minute when its temperature is 75°C.
A. What is the limiting value of the temperature of the coffee?
\[ \lim_{t \to \infty} T(t) = \quad \]
B. What is the limiting value of the rate of cooling?
\[ \lim_{t \to \infty} \frac{dT}{dt} = \quad \]
C. Find the constant \( k \) in the differential equation.
\( k = \quad \)
D. Use Euler’s method with step size \( h = 3 \) minutes to estimate the temperature of the coffee after 15 minutes.
\( T(15) = \quad \)
Correct Answers:
• 25
• 0
• -0.04
• 59.302574592

9. (10 pts) Library/Rochester/setDiffEQ4Linear1stOrder/osu_de_4.15.pg
Find the particular solution of the differential equation
\[ \frac{dy}{dx} + y \cos(x) = 7 \cos(x) \]
satisfying the initial condition \( y(0) = 9 \).
Answer: \( y(x) = \quad \)
Correct Answers:
• \( 7 + 2^e^{-\sin(x)} \)

10. (10 pts) Library/Rochester/setDiffEQ4Linear1stOrder/ur_de_4.16.pg
Find the function satisfying the differential equation
\[ f'(t) - f(t) = -5t \]
and the condition \( f(2) = -5 \).
\( f(t) = \quad \)
Correct Answers:
• \(-2.70670566473225*2.71828182845905^t - 5^t - 5 \)

11. (10 pts) Library/Rochester/setDiffEQ4Linear1stOrder/ur_de_4.15.pg
Solve the initial value problem
\[ \frac{dx}{dt} + 2x = \cos(2t) \]
with \( x(0) = -1 \).
\( x(t) = \quad \)
Correct Answers:
• \(-1.25*2.71828182845905*(-2^t) + 2^8*\cos(2^t) + 2^8*\sin(2^t) \)

12. (10 pts) Library/Rochester/setDiffEQ3Separable/ur_de_3.15.pg
Solve the separable differential equation
\[ \frac{dy}{dx} = \frac{-0.2}{\cos(y)} \]
and find the particular solution satisfying the initial condition
\( y(0) = \frac{\pi}{3} \).
\( y(x) = \quad \)
13. (10 pts) Library/Rochester/setDiffEQ3Separable/jas7_4_5.pg
Find \( u \) from the differential equation and initial condition.

\[
\frac{du}{dt} = e^{1.1t - 1.7u}, \quad u(0) = 2.
\]

\( u = \) ________________.

Correct Answers:

- \( \arcsin(-0.2x + 0.866025403784439) \)

14. (10 pts) Library/Rochester/setDiffEQ3Separable/ns7_4_8a.pg
Solve the separable differential equation for.

\[
\frac{dy}{dx} = \frac{1+x}{xy^{17}}; \quad x > 0
\]

Use the following initial condition: \( y(1) = 2 \).

\( y^{18} = \) ________________.

Correct Answers:

- \( \frac{1}{1.7} \ln(28.4186455019425 + ((1.7/1.1)*2.71828182845905^{1.1})) \)

- \( 18^\ln(x) + 18^x + 262126 \)
WeBWorK assignment number 11.

due 11/25/2008 at 11:59pm MST.

This set has applications of integration, and a few partial fractions problems.

1. (10 pts) 1250Library/set11_Applications_of_Integration-/1220s1p19.pg

A Solid of Revolution. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = 4x^2, x = 1, y = 0, \text{ about the x-axis} \]

Correct Answers:
- 10.0530964914873

2. (10 pts) 1250Library/set11_Applications_of_Integration-/1220s1p21.pg

Work. A force of 6 pounds is required to hold a spring stretched 0.5 feet beyond its natural length. How much work (in foot-pounds) is done in stretching the spring from its natural length to 0.8 feet beyond its natural length?

\[ F = 6, L = 0.5, \text{ about the x-axis} \]

Correct Answers:
- 3.84

3. (10 pts) 1250Library/set11_Applications_of_Integration/p12-1.pg

Find the volume of the solid generated by revolving about the x-axis the region bounded by the upper half of the ellipse

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

and the x-axis, and thus find the volume of a prolate spheroid. Here \( a \) and \( b \) are positive constants, with \( a > b \).

Volume of the solid of revolution: ____

Correct Answers:
- \((4/3) \cdot 3.141592654 \cdot a \cdot (b^2)\)

4. (10 pts) 1250Library/set11_Applications_of_Integration/p12-2.pg

The region bounded by \( y = 2 + \sin x, y = 0, x = 0 \) and \( 2\pi \) is revolved about the y-axis. Find the volume that results.

Hint:

\[ \int x \sin x \, dx = \sin x - x \cos x + C. \]

Volume of the solid of revolution: ____

Correct Answers:
- 208.57195924897

5. (10 pts) 1250Library/set11_Applications_of_Integration/p12-8.pg

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = x^2, y = 1; \text{ about } y = 9 \]

Correct Answers:
- 70.3716754404114

6. (10 pts) 1250Library/set11_Applications_of_Integration/p13-1.pg

Find the volume of the solid formed by rotating the region inside the first quadrant enclosed by

\[ y = x^4, y = 8x \]

about the x-axis.

Correct Answers:
- 357.443430808439

7. (10 pts) 1250Library/set11_Applications_of_Integration/p13-2.pg

A ball of radius 11 has a round hole of radius 8 drilled through its center. Find the volume of the resulting solid.

Correct Answers:
- 1802.6063342226

8. (10 pts) 1250Library/set11_Applications_of_Integration/p13-3.pg

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = 3x^2, x = 1, y = 0, \text{ about the x-axis} \]

Correct Answers:
- 5.65486677646163
9. (10 pts) 1250Library/set11_Applications_of_Integration/p13-4.pg
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = x^2, y = 0, x = 0, x = 3, \text{ about the y-axis} \]

Correct Answers:
• 127.234502470387

10. (10 pts) 1250Library/set11_Applications_of_Integration/p13-6.pg
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = 0, y = x(1 - x) \text{ about the axis } x = 0 \]

Correct Answers:
• 0.523598775598299

11. (10 pts) 1250Library/set11_Applications_of_Integration/p14.pg
arc length
Find the length of the curve defined by

\[ y = 4x^{3/2} + 1 \]
from \( x = 2 \) to \( x = 6 \).

Correct Answers:
• 47.6462469775597

12. (10 pts) 1250Library/set11_Applications_of_Integration/p15.pg
Work
A force of 4 pounds is required to hold a spring stretched 0.2 feet beyond its natural length. How much work (in foot-pounds) is done in stretching the spring from its natural length to 0.7 feet beyond its natural length? ________________

Correct Answers:
• 4.9

13. (10 pts) 1250Library/set12_Further_Techniques_and_Applications_of_Integration/1220s14p1.pg
Use the method of partial fraction decomposition to perform the following integration.

\[ \int \frac{2}{x^2 + 3x} \, dx = \] ________________

Hint: See example 5 in section 8.5 of the text.

Correct Answers:
• \( \frac{2}{3}(\ln(x) - \ln(x+3)) \)

14. (10 pts) 1250Library/set12_Further_Techniques_and_Applications_of_Integration/1220s14p2.pg
Use the method of partial fraction decomposition to perform the following integration.

\[ \int \frac{5x}{2x^3 + 6x^2} \, dx = \] ________________

Hint: See example 5 in section 8.5 of the text.

Correct Answers:
• \( \frac{5}{6}(\ln(x) - \ln(x+3)) \)

15. (10 pts) 1250Library/set12_Further_Techniques_and_Applications_of_Integration/1220s14p3.pg
Use the method of partial fraction decomposition to perform the following integration.

\[ \int \frac{2x^2 - x - 20}{x^2 + x - 6} \, dx = \] ________________

Hint: \( 2x^2 - x - 20 = 2(x^2 + x - 6) - 3x - 8 \)

Correct Answers:
• \( 2x - 1/5(\ln(x+3) + 14*\ln(x-2)) \)
WeBWorK assignment number 12.

due 12/02/2008 at 11:59pm MST.

This set has more applications of integration.

1. (1 pt) 1250Library/set12_Further_Techniques_and_Applications_of_Integration-/1220s14p4.pg

In many population growth problems, there is an upper limit beyond which the population cannot grow. Many scientists agree that the earth will not support a population of more than 16 billion. There were 2 billion people on earth in 1925 and 4 billion in 1975. If \( y \) is the population \( t \) years after 1925, an appropriate model is the differential equation

\[
\frac{dy}{dt} = ky(16 - y).
\]

Note that the growth rate approaches zero as the population approaches its maximum size. When the population is zero then we have the ordinary exponential growth described by

\[
\frac{dy}{dx} = 16ky.
\]

As the population grows it transits from exponential growth to stability.

(a) Solve this differential equation. \( y = \) ____________

(b) The population in 2015 will be \( y = \) ____________ billion.

(c) The population will be 9 billion some time in the year ____________

Note that the data in this problem are out of date, so the numerical answers you’ll obtain will not be consistent with current population figures.

Hint: (a) Separate variables and use the given information to solve for \( y \). (b) Evaluate \( y \). (c) Solve for the appropriate time.

Correct Answers:

- \( 16/(1+7e^{-0.015t}) \)
- 6.34
- 2054

2. (1 pt) 1250Library/set12_Further_Techniques_and_Applications_of_Integration-/1220s14p5.pg

Compute

\[
\int \frac{\sqrt{x}+1}{\sqrt{x}-1} \, dx = \text{__________}.
\]

Hint: Use a substitution and partial fractions.

Correct Answers:

- \( x+4*\sqrt{x}+4*\log(\sqrt{x})-1 \)

3. (1 pt) 1250Library/set12_Further_Techniques_and_Applications_of_Integration-/1220s17p2.pg

If a 1000-pound capsule weighs only 165 pounds on the moon, how much work is done in propelling this capsule out of the moon’s gravitational field? (Note: The radius of the moon is roughly 1080 miles)

The amount of work is ____________ mile-pounds.

Correct Answers:

- \( 1.782 \times 10^{15} \)

4. (1 pt) 1250Library/set12_Further_Techniques_and_Applications_of_Integration-/q1.pg

The solution of the initial value problem

\[
\frac{dy}{dx} = y\sin x - 2\sin x, \quad y(0) = 0
\]

is \( y = \) ____________

Hint: Find an integrating factor.

Correct Answers:

- \( 2-2\exp(1)\exp(-\cos x) \)

5. (1 pt) 1250Library/set12_Further_Techniques_and_Applications_of_Integration-/q2.pg

A lake contains an amount \( V \) liters of water. Initially, the water contains an amount \( P \) liters of a pollutant. Water is flowing into the lake at a rate of \( F \) liters per second, and well mixed lake fluid is flowing out at the same rate. The ratio of pollutant to total fluid in the inflow is \( r \). The amount of \( p(t) \) of pollutant in the lake is

\[
p(t) = \text{__________}
\]

Hint: Set up a linear differential equation and find an integrating factor.

Correct Answers:

- \( \frac{rV+(P-rV)\exp(-Ft/V)}{V} \)

6. (1 pt) 1250Library/set12_Further_Techniques_and_Applications_of_Integration-/q3.pg

The average value of \( \sin x \) in the interval \([0, \pi]\) is ____________.

Hint: Apply the Mean Value Theorem for integrals.

Correct Answers:

- \( 0.636619772367581 \)

7. (1 pt) set12/q4.pg

The center of mass of a quarter circle given by

\[
y = \sqrt{r^2 - x^2}, \quad x \in [0,r]
\]

is the point \( P = ( _____ , _____ ) \).

Hint: Use symmetry.

Correct Answers:

- \( 4\pi/3, 3.14159265358979 \)
- \( 4\pi/3, 3.14159265358979 \)
8. Let $T$ be the triangle with vertices $(a,r)$, $(b,s)$, and $(c,t)$ (and assume constant density). The center of mass of $T$ is $C = (____,____)$. 

**Hint:** You can do this the easy or the hard way.

*Correct Answers:*
- $(a+b+c)/3$
- $(r+s+t)/3$

9. I got the following data from this web page: The diameter of the Sun is 1.392 million kilometers. The mean distance of the Sun from the Earth (measured from center to center) is 149 million kilometers. The surface gravity on the Sun is 274 meters per second squared. Using those numbers, the escape velocity on the surface of the Sun is ____ kilometers per second.

For the next part, let’s first solve a general problem. Suppose you have a planet or star of radius $R$ and surface gravity $G$. Then the escape velocity on the surface of that object is _____.

(We answered that question in class.) Now suppose you are already at a distance $S > R$ from the center of that object. Then the escape velocity is only _____.

Let’s apply this to our actual world. Suppose you are launching a space craft from a point in Earth’s orbit. Ignoring earth’s gravity the minimum speed required to cause your craft to leave our solar system is ____ kilometers per second.

*Correct Answers:*
- 617.582383168432
- $\sqrt{2 GR}$
- $\sqrt{2 G R^2 / S}$
- 42.2091284379331
1. Compute \( \lim_{x \to 1} \frac{2x - \sin x}{x} \).
   
   \[ \lim_{x \to 1} \frac{2x - \sin x}{x} = \frac{2 - \sin(1)}{1} \]
   
   Correct Answers:
   
   \[ 2 - \sin(1) \]

2. Compute \( \lim_{x \to 0} \frac{e^x - e^{-x}}{2 \sin x} \).
   
   \[ \lim_{x \to 0} \frac{e^x - e^{-x}}{2 \sin x} = \frac{1}{1} \]
   
   Correct Answers:
   
   \[ 1 \]

3. Compute \( \lim_{x \to 0} \frac{\sin x}{x^2} - \frac{\tan x}{x^2} \).
   
   \[ \lim_{x \to 0} \frac{\sin x}{x^2} - \frac{\tan x}{x^2} = \frac{1}{2} \]
   
   Hint: Apply l’Hôpital’s Rule repeatedly.
   
   Correct Answers:
   
   \[ 0.5 \]

4. Compute \( \lim_{x \to 0} \frac{\cosh x - 1}{x^2} \).
   
   \[ \lim_{x \to 0} \frac{\cosh x - 1}{x^2} = \frac{0.5}{1} \]
   
   Hint: Apply l’Hôpital’s Rule repeatedly.
   
   Correct Answers:
   
   \[ 0.5 \]

5. Find the indicated limit. Begin by deciding why l’Hôpital’s rule is not applicable. Then find the limit by other means.
   
   \[ \lim_{x \to 0} \frac{x^2 \sin \frac{1}{x}}{x \tan x} = \frac{\sin \frac{1}{x}}{\tan x} = \frac{1}{1} \]
   
   Hint: \( |\sin x| \leq 1 \)
   
   Correct Answers:
   
   \[ 1 \]

6. Compute \( \lim_{x \to \infty} \frac{(\ln x)^2}{2x} \).
   
   \[ \lim_{x \to \infty} \frac{(\ln x)^2}{2x} = \frac{0}{\infty} = 0 \]
   
   Correct Answers:
   
   \[ 0 \]

7. Compute \( \lim_{x \to \infty} \frac{3x}{\ln(100x + e^x)} \).
   
   \[ \lim_{x \to \infty} \frac{3x}{\ln(100x + e^x)} = 3 \]
   
   Correct Answers:
   
   \[ 3 \]

8. Compute \( \lim_{x \to 0} \left( \frac{\cos x}{x^2} \right)^{1/2} \).
   
   \[ \lim_{x \to 0} \left( \frac{\cos x}{x^2} \right)^{1/2} = \frac{1}{\sqrt{e}} \]
   
   Correct Answers:
   
   \[ \frac{1}{\sqrt{e}} \]

9. Compute \( \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^x \).
   
   \[ \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^x = e \]
   
   Correct Answers:
   
   \[ e \]

10. Compute \( \lim_{x \to 0^+} x^4 \).
    
    \[ \lim_{x \to 0^+} x^4 = 0 \]
    
    Correct Answers:
    
    \[ 0 \]

11. Find the area of the region under the curve \( y = \frac{1}{x^2 + x} \) to the right of \( x = 1 \).
    
    The area is \( \ln(2) \).
12. (10 pts) set13/1220s17p3.pg
Suppose that a company expects its annual profits \( t \) years from now to be \( f(t) \) dollars and that interest is considered to be compounded continuously at an annual rate \( r \). Then the present value of all future profits can be shown to be

\[
FP = \int_0^\infty e^{-rt} f(t) \, dt
\]

Find \( FP \) if \( r = 0.08 \) and \( f(t) = 100,000 + 1000t \).

The present value is _______ dollars.

Correct Answers:
- 1406250

13. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s17p4.pg
In electromagnetic theory, the magnetic potential \( u \) at a point on the axis of a circular coil is given by

\[
u = Ar \int_a^\infty \frac{dx}{(r^2 + x^2)^{3/2}}
\]

where \( A, r, a \) are constants. Compute \( u \).  

**Hint:** The integration is a little tricky. Substitute \( x = r \tan \theta \).

Correct Answers:
- \( A/r(1-(a)/\sqrt{r^2+a^2}) \)

14. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s18p1.pg
Evaluate the improper integral if it converges. In this and all other problems of this set, enter the capital letter "D" if it diverges.

\[
\int_1^3 \frac{dx}{(x-1)^{4/3}}
\]

Correct Answers:
- D

15. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s18p2.pg
Evaluate the improper integral.

\[
\int_0^9 \frac{dx}{\sqrt{9-x}}
\]

Correct Answers:
- 6

16. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s18p3.pg
Evaluate the improper integral.

\[
\int_0^1 \frac{x}{\sqrt{1-x^2}} \, dx
\]

Correct Answers:
- .75

17. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s18p4.pg
Evaluate the improper integral.

\[
\int_0^{\pi/2} \frac{\cos x}{\sqrt{\sin x}} \, dx
\]

Correct Answers:
- 1.5

18. (10 pts) set13/1220s18p5.pg
Evaluate the improper integral.

\[
\int_2^4 \frac{dx}{\sqrt{4x-x^2}}
\]

**Hint:** Use the technique of completing the square.

Correct Answers:
- 1.5707963267949

19. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s28p1.pg
Use the Bisection Method to approximate the real root of the given equation on the given interval. Your answer should be accurate to two decimal places.

\[x^4 + 5x^3 + 1 = 0; \quad [1, 2]\]

The root \( r \approx \) _______

Correct Answers:
- -.61

20. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s28p2.pg
Use the Bisection Method to approximate the real root of the given equation on the given interval. Your answer should be accurate to two decimal places.

\[x - 2 + 2\ln x = 0; \quad [1, 2]\]

The root \( r \approx \) _______

Correct Answers:
- 1.37

21. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s28p3.pg
Use Newton’s Method to approximate the root of \( x\ln x = 2 \) accurate to five decimal places. Begin by sketching a graph.

The root \( r \approx \) _______

Correct Answers:
- 2.345751

22. (10 pts) 1250Library/set13_Limits_LHopitals_Rule_and_Numerical_Methods-/1220s28p4.pg
Section 11.3, Problem 6. Use Newton’s Method to approximate the real root of \( 7x^3 + x - 5 = 0 \) accurate to five decimal places. Begin by sketching a graph.

The root \( r \approx \) _______

Correct Answers:
- .84070
23. (10 pts) Compute \( \lim_{x \to 1} \frac{2x - \sin x}{x} = \) ____________

Compute \( \lim_{x \to 1} \frac{\ln x^2}{x^2 - 1} = \) ____________

Correct Answers:

- 2 - \sin(1)
- 1

24. (10 pts) Compute \( \lim_{x \to 0} \frac{e^x - e^{-x}}{2 \sin x} = \) ____________

Section 9.1, Problem 10. Compute

26. (10 pts) Compute \( \lim_{x \to 0} \frac{e^x - e^{-x}}{2 \sin x} = \) ____________

Correct Answers:

- 1

27. (10 pts) Find the indicated limit. Begin by deciding why l’Hopital’s rule is not applicable. Then find the limit by other means.

Find the indicated limit. Begin by deciding why l’Hopital’s rule is not applicable. Then find the limit by other means.

\( \lim_{x \to 0} \frac{x^2 \sin \frac{1}{x}}{\tan x} = \) ____________

Hint: \(|\sin x| \leq 1\).

Correct Answers:

- 0
WeBWorK assignment number 14

Hsiang-Ping Huang
due 12/18/2008 at 11:59pm MST

This is the final homework set for this semester. Its purpose is to review the entire semester and help you prepare for the final exam. You should be able to answer questions like these (but not just these particular questions) on the final exam. So make sure you understand everything surrounding each of the questions on this set. Use them as a guide in reviewing our work this semester.

Remember that the final exam will take place

Friday, December 19, 10:30am-12:30pm,
in our regular classroom. It will be closed books and notes, no electronics, as usual.

The original schedule class states that the deadline for this set will be the last day of classes. The reason for that choice is that I don’t think we should have assignments due during final’s week. On the other hand, our final is on the very last day of that week, and that deadline would be a full week before the final. It occurred to me that you might prefer to work on this set during final’s week. So I set the deadline at midnight on the day before the final. Of course, to be really on top of things you should finish work on this set by Friday, December 12.

Peter Alfeld, JWB 127, 581-6842.

1. (10 pts) 1250Library/set14_Review/1250s14q1.pg

Limits: Compute \( \lim_{x \to -3} \frac{x^2 - 2x + 12}{x - 3} = \) ____________.

Correct Answers:
- -1

2. (10 pts) 1250Library/set14_Review/1250s14q2.pg

The Product Rule: Compute \( D_x (\sin(x))(x^2 - 1) = \) ____________.

Correct Answers:
- \( \cos(x) * x^2 - \cos(x) + 2 * \sin(x) * x \)

3. (10 pts) 1250Library/set14_Review/1250s14q3.pg

The Quotient Rule: Compute \( D_x \frac{\sin x}{x^2 - 1} = \) ____________.

Correct Answers:
- \( \frac{(\cos(x) * x^2 - \cos(x) - 2 * \sin(x) * x)}{(x^2 - 2 * x * \sin(x) + 1)} \)

4. (10 pts) 1250Library/set14_Review/1250s14q4.pg

The Chain Rule: Compute \( D_x \cos^2(\sin(x)) = \) ____________.

Correct Answers:
- \( -2 \cos(\sin(x)) * \cos(x) * \sin(\sin(x)) \)

5. (10 pts) 1250Library/set14_Review/1250s14q5.pg

A word problem:
What number exceeds its square by the maximum amount?
Answer: ____________.

Correct Answers:
- 0.5

6. (0 pts) 1250Library/set14_Review/1250s14q6.pg

Graphing: This question does not actually require an answer since it’s hard in WeBWorK to ask for a graph. But you should be able to draw the graphs of simple functions using symmetry and derivatives. Your graphs should clearly show important features like stationary points, minima, maxima, points of inflection, and asymptotes. Practice until you get it down. For example, draw the graph of

\[ f(x) = \frac{x^2}{x + 1}. \]

7. (10 pts) 1250Library/set14_Review/1250s14q7.pg

Indefinite Integrals: Evaluate the indefinite integral.

\[ \int x^3 \sqrt{1 + x^4} \, dx = \] ____________.

Correct Answers:
- \( \frac{2}{3} * 0.25 * (x^4 + 1)^{3/2} \)

8. (10 pts) 1250Library/set14_Review/1250s14q8.pg

Definite Integrals:
Evaluate \( \int_{5}^{1} (6x^2 - 10x + 7) \, dx = \) ____________.

Correct Answers:
- 156

9. (10 pts) 1250Library/set14_Review/1250s14q9.pg

Areas: There is a line through the origin that divides the region bounded by the parabola \( y = 6x - 2x^2 \) and the x-axis into two regions with equal area. What is the slope of that line?

Correct Answers:
- 1.2377968440954

10. (10 pts) 1250Library/set14_Review/1250s14q10.pg

Trigonometric Integrals: Some trigonometric integrals can be evaluated by repeated integration by parts, by using trigonometric identities or by using suitable trigonometric identities.

\[ \int_{0}^{\pi} \sin^2 x \, dx = \] ____________.

\[ \int \sin^2 x \, dx = \] ____________.

\[ \int \frac{1}{\tan^2 x} \, dx = \] ____________.

Correct Answers:
3.14159265358979
-1/2*cos(x)*sin(x)+x/2
log(sin(x))

11. (10 pts) 1250Library/set14_Review/1250s14q11.pg
Integration by Substitution: Some integrals are obvious, others impossible, and yet others can be handled only with a bit of cleverness. In class we have been referring to the use of the formula
\[ \int f(u(x))u'(x)\,dx \]
as the "obvious" substitution. Look at the integrals below. Remember that in WeBWorK we ignore the integration constant, and for definite integrals you may have to change the limits of integration when doing the integration.

\[ \int e^{\sin(x)} \cos(x)\,dx = \, \text{__________} \]
\[ \int_1^2 x \sin(x^2)\,dx = \, \text{__________} \]
\[ \int \frac{1}{\sin(x)}\,dx = \, \text{__________} \]

Correct Answers:
- \[ \exp(\sin(x)) \]
- 0.596972963365876
- \[ \log(\log(x)) \]

12. (10 pts) 1250Library/set14_Review/1250s14q12.pg
More Integration by Substitution:
Compute these integrals:
\[ \int \frac{x}{\sqrt{3x+4}}\,dx = \, \text{__________} \]
\[ \int_0^\pi \frac{\pi x-1}{\sqrt{x^2+\pi}}\,dx = \, \text{__________} \]

Correct Answers:
- \[ \frac{(2\sqrt{3}\pi+4)(2\pi-8)}{27} \]
- 4.42673805745442

13. (10 pts) 1250Library/set14_Review/1250s14q13.pg
Inverse Trig Functions: The trigonometric functions are not invertible because by the nature of their definition in terms of a point moving around the unit circle there are many angles that give rise to the same function value. However, there are problems where you need to know an angle having a certain property, and to facilitate finding that angle a notion of "inverses" of the trig functions are created by suitably restricting the domain of the trigonometric functions. You should understand the conventions and definitions involved, and you should be able to differentiate the inverses of the trigonometric functions.

\[ \frac{d}{dx} \arcsin(x^2) = \, \text{__________} \]
\[ \frac{d}{dx} \arctan(e^x) = \, \text{__________} \]

You should also be able to use the inverse trigonometric functions to solve trigonometric equations.

For example, if the angle \( x \) is known to be in the interval \([-\pi/4, \pi/4]\) and \( 3 \sin(2x) = 2 \), then
\[ x = \, \text{__________} \]

The negative angle \( \theta \) closest to zero that satisfies \( \cos(\theta) = \frac{3}{7} \) is

Correct Answers:
- \[ \frac{2x}{\sqrt{1-x^4}} \]
- \[ \exp(x)/(1+\exp(2x)) \]
- 0.364863828113483
- -1.12788528272126

14. (10 pts) 1250Library/set14_Review/1250s14q14.pg
Inverse Functions and their derivatives: This section builds up to defining the exponential as the inverse function of the logarithm.

Suppose \( f(x) = 3x + 4 \). Then the inverse of \( f \) is given by
\[ f^{-1}(x) = \, \text{__________} \]

Moreover,
\[ f'(x) = \, \text{__________} \]
\[ (f^{-1})'(x) = \, \text{__________} \]

In general, if \( f'(x) = A \), then
\[ (f^{-1})'(f(x)) = \, \text{__________} \] (Note: Your answer must be in terms of \( A \)).

Now let \( f(x) = x + x^5 \). Then
\[ f(1) = \, \text{__________} \]
\[ f'(1) = \, \text{__________} \]
\[ f^{-1}(2) = \, \text{__________} \]
\[ (f^{-1})'(2) = \, \text{__________} \]

Notice that you can obtain the last two results without knowing a general expression for the inverse function of \( f \). Indeed, if you feel enterprising try to come up with such an expression. Let me know what you find.

Correct Answers:
- \( (x-4)/3 \)
- 3
- 0.33333333333333
- 1/\( A \)
- 2
- 6
- 1
- 0.166666666666667

15. (10 pts) 1250Library/set14_Review/1250s14q15.pg
Logarithms: Compute
\[ \frac{d}{dx} \ln(x^2+1) = \, \text{__________} \]
and
\[ \int \frac{\cos(x)}{\sin(x)}\,dx = \, \text{__________} \]

Correct Answers:
- \[ 2x/\left(x^2+1\right) \]
- \[ \ln(2 + \sin(x)) \]
16. (10 pts) Exponentials and Logarithms: The (natural) exponential equals its own derivative and its own antiderivative. Once you accept that the derivative of the natural logarithm is 1/x these facts flow from the fact that the exponential is the inverse of the natural logarithm. Both exponentials and logarithms appear frequently in applications.

Here are some exercises reinforcing the unique status of the exponential.

\[ \frac{d}{dx} e^x = \text{__________}. \]
\[ \frac{d}{dx} e^{\sin x} = \text{__________}. \]
\[ \int x e^{x^2} \, dx = \text{__________}. \] (As usual ignore the integration constant for indefinite integrals in this homework.)

Correct Answers:
- \(2^x \log(2)\)
- \(e^{\sin(x)} \cos(x)\)
- \(\exp(x^2)/2\)
- \((x^{\sqrt{x}} (\log(x) + 2))/(2 \cdot \sqrt{x})\)

17. (10 pts) Exponential Growth and Decay: Here are a couple of examples for applications of exponentials and logarithms.

You find out that in the year 1800 an ancestor of yours invested 100 dollars at 6 percent annual interest, compounded yearly. You happen to be her sole known descendant and in the year 2005 you collect the accumulated tidy sum of ______ dollars. You retire and devote the next 10 years of your life to writing a detailed biography of your remarkable ancestor.

Strontium-90 is a biologically important radioactive isotope that is created in nuclear explosions. It has a half-life of 28 years. To reduce the amount created in a particular explosion by a factor 1,000 you would have to wait ______ years. Round your answer to the nearest integer.

Seeds found in a grave in Egypt proved to have only 66% of the Carbon-14 of living tissue. Those seeds were harvested ______ years ago. The half life of Carbon-14 is 5,730 years.

Correct Answers:
- 15406443
- 279
- 3434.91766348523

18. (10 pts) The Hyperbolic Functions and their Inverses: For our purposes, the hyperbolic functions, such as
\[ \sinh x = \frac{e^x - e^{-x}}{2} \] and \[ \cosh x = \frac{e^x + e^{-x}}{2} \]
are simply extensions of the exponential, and any questions concerning them can be answered by using what we know about exponentials. They do have a host of properties that can become useful if you do extensive work in an area that involves hyperbolic functions, but their importance and significance is much more limited than that of exponential functions and logarithms.

Let \(f(x) = \sinh x \cosh x\).

\[ \frac{d}{dx} f(x) = \text{__________}. \]
\[ f'(x) = \text{__________}. \]
\[ f^{-1}(x) = \text{__________}. \]

\(f^{-1}(x) = __________). \)

Correct Answers:
- \((\exp(2 \cdot x) + \exp(-2 \cdot x))/2\)
- \((\exp(2 \cdot x) + \exp(-2 \cdot x))/8\)
- \(1/\sqrt{4 \cdot x^2 + 1}\)

19. (10 pts) Integration by Parts: This is the most important integration technique we’ve discussed in this class. It has a wide range of applications beyond increasing our list of integration rules.

\[ \int z^3 \ln z \, dz = \text{__________}. \]
\[ \int e^t \cos t \, dt = \text{__________}. \]
\[ \int 2 \pi \sin(x) \sin(x+1) \, dx = \text{__________}. \]

Correct Answers:
- \(z^4 \ln(z + 1)/4\)
- \(1/2 \exp(t) (\cos(t) + \sin(t))\)
- \(1.69740975483297\)

20. (10 pts) Integration by Partial Fractions: This technique involves an unusual decomposition of rational expressions: instead of factoring them you write them as sums of simpler terms.

\[ \int \frac{3}{x^3 - 1} \, dx = \text{__________}. \]
\[ \int \frac{x}{x^2 - 3x - 4} \, dx = \text{__________}. \]

\[ \int \frac{1}{x^2 - 16} \, dx = \text{__________}. \]

Correct Answers:
- \((3 \cdot \log(x - 1) - \log(x + 1))/2\)
- \(-2 \cdot \log(x - 1) + 3 \cdot \log(x + 4)\)
- \((-2 \cdot \text{atan}(x/2) + \log(x - 2) - \log(x + 2))/32\)
21. (10 pts) Use the Fundamental Theorem of Calculus to find the derivative of 

\[ f(x) = \int_4^x \left( \frac{1}{4} t^2 - 1 \right)^5 dt \]

**Correct Answers:**
- \( 2 \cdot (1/4 \cdot x^4 - 1)^5 \cdot x \)

22. (10 pts) Find the length of the curve defined by 

\[ y = 4 \cdot x^{3/2} - 1 \]

from \( x = 4 \) to \( x = 7 \).

**Correct Answers:**
- 42.1885210626592

23. (10 pts) A ball of radius \( R \) has a round hole of radius \( r \) drilled through its center. Find the volume of the resulting solid.

**Correct Answers:**
- \( \frac{4}{3} \pi R^3 \left( R^2 - r^2 \right)^{3/2} \)

24. (10 pts) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = ax^n, x = 1, y = 0, \text{ where } n \geq 1 \text{ and } a > 0, \text{ about the } x\text{-axis} \]

**Correct Answers:**
- \( a \sqrt[3]{1/2} \pi \cdot \left( (R^2) - (r^2) \right)^{3/2} \)

25. (10 pts) Improper Integrals, Infinite Integrands: Compute the improper integrals below. Enter the letter "D" if they diverge.

\[ \int_1^\infty \frac{1}{x^2} \, dx = \]

\[ \int_1^\infty \frac{1}{1 + x^2} \, dx = \]

\[ \int_1^\infty e^x \, dx = \]

**Correct Answers:**
- 1
- D
- 0.36787944117142
- D
- 3.14159265358979
- 23.1406926327793

26. (10 pts) Indeterminate Expressions of the Form \( 0/0 \): Find the indicated limits. Make sure you have an Indeterminate Expression before you apply the Rule of L'Hopital. Enter the letter "D" if the limit does not exist.

\[ \lim_{x \to 0} \frac{\sin x}{x} = \]

\[ \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \]

\[ \lim_{x \to 0} \frac{e^x - e^{-x}}{\sin(x)} = \]

**Correct Answers:**
- 1
- D
- 0.5
- 2
- D

27. (10 pts) Improper Integrals, Infinite Limits of Integration: Integrate may have infinite limits of integration, or integrands that have singularities. Such integrals are called "improper" even though there is nothing wrong with such integrals. They may have well defined values, in which case we say they converge, or they may not, in which case we say they diverge. In this and the next problem, give the value of the integral if it converges, and enter the letter "D" if it diverges.

\[ \int_2^\infty \frac{1}{x^2} \, dx = \]

\[ \int_2^\infty \frac{1}{x^2} \, dx = \]

\[ \int_2^\infty e^{-x} \, dx = \]

**Correct Answers:**
- 1
- 0.5
- 2
- D

4
**28. (10 pts) 1250Library/set14_Review/1250s14q28.pg**

**Other Indeterminate Expressions:** Find the indicated limits. You may have to manipulate your expression before applying the Rule of L’Hospital. Enter the letter "D" if the limit does not exist.

Suppose \( p(x) \) is a polynomial of degree greater than 0. Then

\[
\lim_{x \to \infty} \frac{p(x)}{e^x} = \text{______} \quad \text{and} \quad \lim_{x \to \infty} \frac{p(x)}{\ln(x)} = \text{______}.
\]

\[
\lim_{x \to 0} (1 + x)^{1/x} = \text{______}.
\]

\[
\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = \text{______}.
\]

\[
\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x} = \text{______}.
\]

\[
\lim_{x \to 0^+} x^x = \text{______}.
\]

\[
\lim_{x \to 0^+} x^x = \text{______}.
\]

**Correct Answers:**

- 0
- 0
- 2.71828182845905
- 2.71828182845905
- 7.38905609893065
- 1
- 0

**29. (10 pts) 1250Library/set14_Review/1250s14q29.pg**

**Sequences:** A sequence is of the form 

\[ a_1, a_2, a_3, a_4, \ldots \]

where the \( a_n \) are real numbers. Technically, a sequence is a function whose domain is the set of natural numbers, and whose range is a subset of the real numbers. Sequences may be defined in various ways:

By listing, and appealing (via the three dots) to your intuition. Suppose the sequence is

\[ 1, 2, 3, 4, 5, 6, 2, 5, 10, 17, 26, 37, \ldots \]

Then the \( n \)-th term is

\[ a_n = \text{______}. \]

**Explicitly.** For example, suppose \( a_n = n^n \). Then \( a_1 = \text{______}, \) \( a_2 = \text{______}, \) and \( a_3 = \text{______}. \)

**Recursively.** For example, the *Fibonacci Sequence* is defined by

\[ a_1 = a_2 = 1, \quad a_{n+1} = a_n + a_{n-1}, \quad n = 2, 3, 4, \ldots \]

Thus \( a_3 = \text{______}, \) \( a_4 = \text{______}, \) and \( a_5 = \text{______}. \)

A sequence may or may not have a limit. For the following sequences, enter the limit, or enter the letter "D" if the sequence diverges.

\[ a_n = n \text{ ___}. \]

\[ a_n = \frac{1}{n} \text{ ___}. \]

\[ a_n = \frac{n^2 + 4n - 5}{(2n - 1)(3n - 1)} \text{ ___}. \]

**Correct Answers:**

- \( n/(n^2+1) \)
- 1
- 4
- 27
- 2
- 3
- 5
- 0
- 0
- 0.166666666666667

**30. (10 pts) set14/1250s14q30.pg**

**Series:** A Series (Or Infinite Series) is obtained from a sequence by adding the terms of the sequence. Another sequence associated with the series is the sequence of partial sums. A series converges if its sequence of partial sums converges. The sum of the series is the limit of the sequence of partial sums.

For example, consider the geometric series defined by the sequence

\[ a_n = \frac{1}{r^n}, \quad n = 0, 1, 2, \ldots \]

Then the \( n \)-th partial sum \( S_n \) is given by

\[ S_n = \sum_{k=0}^{n} \frac{1}{r^k} = \text{______} \quad \text{and, for } -1 < 1/r < 1, \]

\[ \sum_{k=0}^{\infty} \frac{1}{r^k} = \lim_{n \to \infty} S_n = \text{______}. \]

Thus

\[ \sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k = \text{______} \quad \text{and} \quad \sum_{k=2}^{\infty} \left(\frac{2}{3}\right)^k = \text{______}. \]

Another situation in which we can actually compute the partial sums occurs if those sums are collapsing. It may not be obvious that that is the case, but look for it in this example:

\[ \sum_{k=0}^{\infty} \frac{1}{(k^2 + 9k + 20)} = \text{______} \quad \text{and} \quad \sum_{k=0}^{\infty} \frac{1}{(k^2 + 9k + 20)} = \text{______}. \]

**Correct Answers:**

- \((1-(1/r))^{\ast(n+1)}/(1-(1/r))\)
- \(1/(1-(1/r))\)
- 2.7519383988411
- 1.11531862151653
- 1/4-1/(n+5)
31. (10 pts) 1250Library/set14/Review/1250s14q31.pg

**Taylor and MacLaurin Series:** Consider the approximation of the exponential by its third degree Taylor Polynomial:
\[ e^x \approx P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}. \]

Compute the error \( e^x - P_3(x) \) for various values of \( x \):
- \( e^0 - P_3(0) = \) ________.
- \( e^{0.1} - P_3(0.1) = \) ________.
- \( e^{0.5} - P_3(0.5) = \) ________.
- \( e^1 - P_3(1) = \) ________.
- \( e^2 - P_3(2) = \) ________.
- \( e^{-1} - P_3(-1) = \) ________.

**Correct Answers:**
- 0
- 4.2514089810392E-06
- 0.00288793736679486
- 0.0516151617923784
- 1.0557276559732
- 0.034546107838109

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32. (10 pts) 1250Library/set14/Review/1250s14q32.pg

**Power Series:** Compute the convergence set for the following power series. Enter "1" or "I" (without the quotation marks) as appropriate. If the convergence radius is infinite enter "1" or "I" for minus infinity and plus infinity, respectively.

For example (see Example 2 on page 459), the series
\[ \sum_{n=0}^{\infty} \frac{x^n}{(n+1)2^n} \]
converges on the interval \([-2, 2]\), so you’d enter -2, I, 2. Try this now:

\[ \sum_{n=0}^{\infty} \frac{x^n}{(n+1)2^n} \] converges for ________.

Similarly answer the following questions:

\[ \sum_{n=1}^{\infty} \frac{(x-1)^n}{n} \] converges for ________.
\[ \sum_{n=1}^{\infty} \frac{(x-1)^n}{n!} \] converges for ________.
\[ \sum_{n=17}^{\infty} \frac{(x+1)^n}{3^n} \] converges for ________.

**Correct Answers:**
- 0
- 2
- 0.034546107838109
- 0.00288793736679486
- 0.0516151617923784
- 1.0557276559732
- 4.2514089810392E-06
- 0
- 1
- 0.5
- 0.16666666666666667
- 0.0416666666666667
- 1
- 0
- 1
- 0.5
- 0.3333333333333333
- 1
- 30
- 35
- 19
- 4

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33. (10 pts) 1250Library/set14/Review/1250s14q33.pg

**Taylor and MacLaurin Series:** Compute the Taylor Series below. WebWorK does not understand factorials, so you will need to enter numerical values. For example, enter "1/6" instead of "1/3".

\[ e^x = \ldots + x + x^2 + x^3 + x^4 + \ldots \]
\[ \cos^2 x = \ldots + x + x^2 + x^3 + x^4 + \ldots \]
\[ x^3 = \ldots + x(x-1) + (x-1)^2 + (x-1)^3 + \ldots \]
\[ 4x^4 + 3x^3 + 2x^2 + x + 1 = \ldots + x(x-1) + (x-1)^2 + (x-1)^3 + \ldots \]

**Correct Answers:**
- 1
- 1
- 0.5
- 0.16666666666666667
- 0.0416666666666667
- 1
- 0
- -1
- 0
- 0.3333333333333333
- 1
- 30
- 35
- 19
- 4

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34. (10 pts) 1250Library/set14/Review/1250s14q34.pg

**Differential Equations:** A differential equation is an equation that involves and unknown function and some of its derivatives. Differential Equations are a large subject on which we teach several courses. In Calculus you only get a first exposure to them and you see some very simple examples. Set up and solve a differential equation to solve the following problem:

You have a pond with decorative fish in your back yard. The pond holds 800 gallons of water. Once a week you pour fresh water into the pond at the rate of 100 gallons per hour. The pond
is filled to the brim, and so as you pour water into the tank wa-
ter flows out at the same rate. There is a pump in the pond that
keeps the water perfectly mixed. Your goal is to add water un-
til any pollutants in the pond are reduced by a factor 1/2. You
keep the fresh water flowing for ________ hours. (Remember
that ww expects your answer to be accurate within one tenth of
one percent. You may want to enter a mathematical expression
rather than a numerical value.)

Correct Answers:
- $5.5451774447956$

35. (10 pts) 1250Library/set14_Review/1250s14q35.pg

Newton’s Method: As discussed in class, to solve the equation

$$f(x) = 0$$

by Newton’s Method we start with a good initial guess $x_0$ and
then run the iteration

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

until we get an approximation $x_{n+1}$ that is good enough for our
purposes.

Suppose you want to compute the cube root of 4 by solving
the equation

$$x^3 - 4 = 0.$$  

Since $1^3 = 1$ and $2^3 = 8$ Let’s start with

$$x_0 = 1.5$$

Then

$$x_1 = \__, \quad x_2 = \__, \quad x_3 = \__, \quad \text{and} \quad x_4 = \__.$$  

To check your answer compute $x_3^3 = \__$.

Enter $x_1$, $x_2$ and $x_3$ with at least 6 correct digits beyond the dec-
imal point.

Correct Answers:
- 1.59259259259259
- 1.58741795698709
- 1.587401052
- 4

36. (10 pts) 1250Library/set14_Review/1250s14q36.pg

More on Newton’s Method: The equation

$$10(x - 1)(x - 2)(x - 3) = 1$$

has three real solutions

$$a < b < c$$

where

$$a = \__, \quad b = \__, \quad \text{and} \quad c = \__.$$  

Enter your answers with at least six correct digits beyond the dec-
imal point.

Hint: Ask what the solutions are if the right hand side is 0 in-
stead of 1, and use Newton’s Method.

Correct Answers:
- 1.05435072607641
- 1.89896874211899
- 3.0466805318046