Lines, Tangents, Derivatives
This assignment will cover the material from Chapters 1.1 – 1.8 and Polynomial Calculus Notes sections 1 – 3(b).

1. (1 pt) a. Find the slope of the line passing through the points (7,8) and (10,6).
   b. Find the slope of the line passing through the points (0,0) and (2,0).

2. (1 pt) Find the equation of the line passing through the point (8,3) with slope -9.
   \[ y = \ldots \]

3. (1 pt) The equation of the line passing through the point (4, -8) which is perpendicular to the line given by the equation \( 3x + 3y = 1 \) is \( y = Ax + B \) where \( A = \ldots \)
   \( B = \ldots \)

4. (1 pt) Find the equation of the line passing through the point (-5, -4) and parallel to the line passing through (-3,0) and (-10,2).
   \[ y = \ldots \]

5. (1 pt) Find the equation of the line which bisects the line segment from (0,0) to (6,2) at right angles.
   \[ y = \ldots \]

6. (1 pt) Find the derivative of \( f(x) = 10x - 10. \)
   \[ f'(x) = \ldots \]

7. (1 pt) Find the equation of the line tangent to the curve \( y = 10x^2 - 5x + 4 \) at the point \( (2,34) \).

8. (1 pt) Find the derivative of \( f(x) = x^8 - 9x^6 + 3x. \)
   \[ f'(x) = \ldots \]

9. (1 pt) Find the slope of the curve \( y = 3x^3 - 2x^2 \) at the point \( (1,1) \).
   \( m = \ldots \)

10. (1 pt) For what values of \( x \) does the curve \( y = x^2 - 2x + 3 \) have:
    Positive slope? \( \ldots \)
    Negative slope? \( \ldots \)
    Zero slope? \( x = \ldots \)
    Your answer to parts 1 and 2 should be an interval \((a, b)\). Use \( \text{INF} \) for \(+\infty\), \(-\text{INF}\) for \(-\infty\).

11. (1 pt) A ball is thrown straight up so its height \( t \) seconds later is \( -16t^2 + 32t + 6. \)
    a. Find the velocity of the ball at \( t \) seconds after it is thrown.
    \[ v = \ldots \]
    b. At what time does the ball reach its maximum height?
    \( t = \ldots \)
    c. Find the acceleration of the ball at any time \( t. \)
    \( a = \ldots \)

12. (1 pt) If \( f(x) = 7x^2 - 8x - 11, \) find \( f'(x). \)
    \[ f'(x) = \ldots \]

13. (1 pt) If \( f(x) = (7x^2 - 4)(5x + 3), \) find \( f'(x). \)
    \[ f'(x) = \ldots \]
    [NOTE: Your answer should be a function in terms of the variable ‘\( x \)’ and not a number!]

14. (1 pt) Find the slope of the curve \( y = 4x^3 - 8x^2 + 6x + 6 \) at the point where \( x = 3. \)
    Slope at \( x = 3 \): \( \ldots \)
WeBWorK demonstration assignment

The main purpose of this WeBWorK set is to familiarize yourself with WeBWorK.

Here are some hints on how to use WeBWorK effectively:

- **After first logging into WeBWorK change your password.**
- Find out how to print a hard copy on the computer system that you are going to use. Print a hard copy of this assignment.
- Get to work on this set right away and answer these questions well before the deadline. Not only will this give you the chance to figure out what’s wrong if an answer is not accepted, you also will avoid the likely rush and congestion prior to the deadline.
- The primary purpose of the WeBWorK assignments in this class is to give you the opportunity to learn by having instant feedback on your active solution of relevant problems. Make the best of it!

1. (1 pt)
Evaluate the expression
\[ 8(6 - 1) = \text{________}. \]

2. (1 pt)
Evaluate the expression
\[ \frac{2}{(5 + 5)} = \text{________}. \]
Enter your answer as a decimal number listing at least 4 decimal digits. (WeBWorK will reject your answer if it differs by more than one tenth of 1 percent from what it thinks the answer is.)

3. (1 pt) Let \( r = 5 \).
Evaluate \( \frac{4}{\pi \times r} = \text{________} \).
Next, enter the expression \( \frac{4}{\pi \times r} = \text{________} \) and let WeBWorK compute the result.

4. (1 pt) Enter here __________ the expression \( \frac{1}{a + \frac{1}{b}} \).

5. (1 pt) Enter here __________ the expression
\[ \frac{a + 1}{2 + b} \]

Enter here __________ the expression
\[ \frac{a + b}{c + d} \]
If WeBWorK rejects your answer use the preview button to see what it thinks you are trying to tell it.

6. (1 pt) Enter here __________ the expression
\[ \sqrt{a + b} \]

Enter here __________ the expression
\[ \frac{a}{\sqrt{a + b}} \]

Enter here __________ the expression
\[ \frac{a + b}{\sqrt{a + b}} \]

7. (1 pt)
Enter here __________ the expression
\[ \sqrt{x^2 + y^2} \]
Enter here __________ the expression
\[ x \sqrt{x^2 + y^2} \]
Enter here __________ the expression
\[ \frac{x + y}{\sqrt{x^2 + y^2}} \]

8. (1 pt) Enter here __________ the expression
\[ \frac{-b + \sqrt{b^2 - 4ac}}{2a} \]
Note: this is an expression that gives the solution of a *quadratic equation* by the *quadratic formula*.
General power rule derivatives and anti-derivatives

This assignment will cover the material from the Polynomial Calculus Notes sections 4 – 5(b) and Page 142, Theorem A, of the text.

1. (1 pt) If \( f(x) = 7 + \frac{8}{x} + 4x^2 \), find \( f'(x) \).

2. (1 pt) Find the \( x \) coordinate of the point on the curve \( y = x + 7x^{-1}, x > 0 \) where the tangent line has slope -8.

3. (1 pt) If \( f(t) = \frac{15}{t^7} \), find \( f'(t) \).

4. (1 pt) Find \( f''(1) \).

5. (1 pt) Find the \( x \) coordinate of the point on the curve \( y = 7x^2 + 7x + 6 \) where the tangent line is perpendicular to the line \( x + 7y = 10 \).

6. (1 pt) Consider the function \( f(x) = 6x^3 - 5x^2 + 2x - 9 \).

An antiderivative of \( f(x) \) is \( F(x) = Ax^4 + Bx^3 + Cx^2 + Dx \)

where \( A \) is _____ and \( B \) is _____ and \( C \) is _____ and \( D \) is _____

7. (1 pt) Consider the function \( f(x) = 7x^{10} + 6x^7 - 4x^3 - 9 \).

Enter an antiderivative of \( f(x) \)

8. (1 pt) Consider the function \( f(x) = \frac{10}{x^3} - \frac{6}{x^7} \).

Let \( F(x) \) be the antiderivative of \( f(x) \) with \( F(1) = 0 \).

Then \( F(x) = \) _____

9. (1 pt) A car traveling at 49 ft/sec decelerates at a constant 6 ft/sec/sec. How many feet does the car travel before coming to a complete stop?
Limits and Continuity
This assignment will cover the material from Chapters 2.1 – 2.10.

1. (1 pt) Evaluate the limit
   \[ \lim_{x \to 0} \frac{\sin 5x}{\sin 2x} \]

2. (1 pt) Evaluate the limit
   \[ \lim_{x \to 0} \frac{\tan x}{2x} \]

3. (1 pt) Evaluate the limit
   \[ \lim_{t \to 1} \frac{t^3 - t}{t^2 - 1} \]

4. (1 pt) Evaluate the limit
   \[ \lim_{a \to 1} \frac{a^3 - 1}{a^2 - 1} \]

5. (1 pt) Evaluate the limit
   \[ \lim_{s \to 36} \frac{36 - s}{6 - \sqrt{s}} \]

6. (1 pt) Evaluate the limit
   \[ \lim_{b \to 1} \frac{\frac{1}{b} - 1}{b - 1} \]

7. (1 pt) Evaluate the limit
   \[ \lim_{b \to 19} \frac{|b + 19|}{b + 19} \]

8. (1 pt) Let \( f(x) = x + 7 \) if \( x \leq 5 \) and \( f(x) = 7 \) if \( x > 5 \).
   Sketch the graph of this function for yourself and find following limits if they exist (if not, enter N).
   \[ \text{____1. } \lim_{x \to 5^-} f(x) \]
   \[ \text{____2. } \lim_{x \to 5^+} f(x) \]
   \[ \text{____3. } \lim_{x \to 5} f(x) \]

9. (1 pt) Let \( f(x) = 6 \) if \( x > 9 \), \( f(x) = 3 \) if \( x = 9 \), \( f(x) = -x + 14 \) if \( -9 \leq x < 9 \), \( f(x) = 23 \) if \( x < -9 \).
   Sketch the graph of this function and find following limits if they exist (if not, enter DNE).
   \[ \text{____1. } \lim_{x \to 9^-} f(x) \]
   \[ \text{____2. } \lim_{x \to 9^+} f(x) \]
   \[ \text{____3. } \lim_{x \to 9} f(x) \]
   \[ \text{____4. } \lim_{x \to -9^-} f(x) \]
   \[ \text{____5. } \lim_{x \to -9^+} f(x) \]
   \[ \text{____6. } \lim_{x \to -9} f(x) \]

10. (1 pt) Evaluate the limit
    \[ \lim_{h \to 0} \frac{4(3+h)^2 + 5(3+h) - (4 \cdot 3^2 + 5 \cdot 3)}{h} \]

11. (1 pt) If an arrow is shot straight upward on the moon with a velocity of 61 m/s, its height (in meters)
    after \( t \) seconds is given by \( h = 61t - 0.83t^2 \).
    What is the velocity of the arrow (in m/s) after 8 seconds? ______
    After how many seconds will the arrow hit the moon? ______
    With what velocity (in m/s) will the arrow hit the moon? ______

12. (1 pt) The slope of the tangent line to the parabola \( y = 2x^2 + 4x + 7 \) at \( x_0 = -3 \) is: ______
    The equation of this tangent line can be written in the form \( y - y_0 = m(x - x_0) \) where \( y_0 \) is: ______

13. (1 pt) For what value of the constant \( c \) is the function \( f \) continuous on \( (-\infty, \infty) \) where
    \[ f(a) = \begin{cases} 
      a^2 - c & \text{if } a \in (-\infty, 5) \\
      ca + 2 & \text{if } a \in [5, \infty) 
    \end{cases} \]

14. (1 pt) The function \( f \) is given by the formula
    \[ f(x) = \frac{-2x^3 - 3x^2 + 15x - 20}{x + 4} \]
    when \( x < -4 \) and by the formula
    \[ f(x) = -3x^2 + 2x + a \]
    when \( -4 \leq x \).
What value must be chosen for \( a \) in order to make this function continuous at -4?

\[
a = 15.
\]

15.(1 pt)
Evaluate the following limits. If needed, enter INF for \( \infty \) and MINF for \( -\infty \).

(a) \[
\lim_{x \to -\infty} \frac{10x + 4}{7x^2 - 6x + 7}
\]

(b) \[
\lim_{x \to -\infty} \frac{10x + 4}{7x^2 - 6x + 7}
\]

16.(1 pt)
Evaluate the following limits. If needed, enter INF for \( \infty \) and MINF for \( -\infty \).

(a) \[
\lim_{x \to \infty} \frac{\sqrt{3 + 6x^2}}{6 + 9x}
\]

(b) \[
\lim_{x \to -\infty} \frac{\sqrt{3 + 6x^2}}{6 + 9x}
\]

17.(1 pt)
Evaluate the following limits. If needed, enter INF for \( \infty \) and MINF for \( -\infty \).

(a) \[
\lim_{x \to \infty} \left( \sqrt{x^2 - 1x + 1} - x \right)
\]

(b) \[
\lim_{x \to -\infty} \left( \sqrt{x^2 - 1x + 1} - x \right)
\]

18.(1 pt)
Evaluate the following limits. If needed, enter INF for \( \infty \) and MINF for \( -\infty \).

(a) \[
\lim_{x \to 4^+} \frac{-19x}{11 - 2x}
\]

(b) \[
\lim_{x \to 4^-} \frac{-19x}{11 - 2x}
\]
Methods of Differentiation, Notation
(aka: product rule, quotient rule and chain rule for differentiation. Differentiation of trigonometric functions.)
This assignment will cover the material from Chapters 3.1 – 3.5.
1. (1 pt) If \( f(x) = \frac{\sqrt{x} - 2}{\sqrt{x} + 2} \)
find \( f'(x) \).

Find \( f'(3) \).

2. (1 pt) If \( f(x) = \frac{2x + 3}{4x + 2} \), find \( f'(x) \).

Find \( f'(4) \).

3. (1 pt) Let \( f(x) = \frac{1 - 4x}{1 + 4x} \). Then \( f'(3) \) is
and \( f''(3) \) is
and \( f'''(3) \) is

4. (1 pt) The angle of elevation to the top of a building is found to be 12° from the ground at a distance of 6000 feet from the base of the building. Find the height of the building.

5. (1 pt) A survey team is trying to estimate the height of a mountain above a level plain. From one point on the plain, they observe that the angle of elevation to the top of the mountain is 28°. From a point 2000 feet closer to the mountain along the plain, they find that the angle of elevation is 32°. How high (in feet) is the mountain?

6. (1 pt) If \( f(x) = \frac{4\sin x}{3 + \cos x} \)
find \( f'(x) \).

Find \( f'(2) \).

7. (1 pt) If \( f(x) = \frac{4\tan x}{x} \), find \( f'(x) \).

Find \( f'(4) \).

8. (1 pt) If \( f(x) = \frac{\tan x - 4}{\sec x} \)
find \( f'(x) \).

Find \( f'(3) \).

9. (1 pt) Let \( f(x) = 10\sin x\cos x \)
\( f'(\frac{\pi}{2}) = \)

10. (1 pt) If \( f(x) = \sin(x^3) \), find \( f'(x) \).

Find \( f'(4) \).

11. (1 pt) Let \( f(x) = \frac{4\cos(x) + 6\sin(x)}{\cos(x)} \). Find \( f'(x) \).

\( f'(x) = \)

12. (1 pt) A traveler is moving from left to right along the curve \( y = x^2 \). When she shuts off the engines, she will continue traveling along the tangent line at the point where she is at that time. At what point should she cut off the engines in order to reach the point (9.5, 70)?

\( x = \)

13. (1 pt) Let \( f(x) = \frac{x^2 + 10}{x + 10} \). Then \( f'(x) = \)

14. (1 pt) Find the equation of the tangent line to the curve \( y = (x + 1)(x^2 - 1) \) at the point (1, 0).

\( y = \)

15. (1 pt) A point is rotating about the circle of radius 1 in the counterclockwise direction. It takes 4.3 minutes to make one revolution. Assuming it starts on the positive x-axis, what are the coordinates of the point in 7.6 minutes?

\( x = \)
\( y = \)

16. (1 pt) Let \( f(x) = (\cos(3x) + 6)^2 \). Find \( f'(x) \).

\( f'(x) = \)
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<th>Question</th>
<th>Description</th>
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<td><strong>17.</strong></td>
<td>(1 pt) Let ( y = (\sin(2x) + 1)^3 ). What is the slope of the tangent line to the curve when ( x = \frac{\pi}{6} )?</td>
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<tr>
<td><strong>18.</strong></td>
<td>(1 pt) Let ( f(x) = (x^2 - 8)^2 ). For what values of ( x ) is ( f''(x) = 0 )? Write the answers in increasing order.</td>
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<tr>
<td><strong>19.</strong></td>
<td>(1 pt) Let ( f(x) = \sqrt{5x^2 + 5x + 5} ) ( f'(x) = ) ( f'(5) = )</td>
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<td><strong>20.</strong></td>
<td>(1 pt) If ( f(x) = \sin(x^5) ), find ( f'(x) ).</td>
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<td><strong>21.</strong></td>
<td>(1 pt) Let ( f(x) = -3\cos^5 x ) ( f'(x) = ) ( f'(5) = )</td>
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<tr>
<td><strong>22.</strong></td>
<td>(1 pt) Let ( f(x) = \cos(7x + 3) ) ( f'(x) = )</td>
</tr>
<tr>
<td><strong>23.</strong></td>
<td>(1 pt) Let ( f(x) = x^{1/3}(2x + 3)^{1/2} ) ( f'(x) = ) ( f'(3) = )</td>
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WeBWorK Assignment 5 due 6/27/06 at 10:59 PM

Higher derivatives, implicit differentiation, related rates and differentials.

This assignment will cover the material from Chapters 3.6 – 3.11.

1. (1 pt) Let \( y = 3x^2 \). Find the change in \( y \), \( \Delta y \) when \( x = 3 \) and \( \Delta x = 0.4 \).

2. (1 pt) Use linear approximation, i.e. the tangent line, to approximate \( \sqrt[3]{1.3} \) as follows:
   \( f(x) = \sqrt[3]{x} \). The equation of the tangent line to \( f(x) \) at \( x = 1 \) can be written in the form \( y = mx + b \) where \( m = \) _______ and where \( b = \) _______.
   Using this, we find our approximation for \( \sqrt[3]{1.3} \) is _______.

3. (1 pt) Let \( f(x) = \frac{x+9}{x+5} \). Then \( f'(x) = \) _______ and \( f''(x) = \) _______.

4. (1 pt) Let \( f(x) = (x+5)(x^2-8) \). For what value of \( x \) is \( f''(x) = 0 \)?

5. (1 pt) Find the slope of the tangent line to the curve \( 0x^2 + 2xy + 4y^3 = -1344 \) at the point \((-2, -7)\).

6. (1 pt) Use implicit differentiation to find the slope of the tangent line to the curve \( \frac{y}{x-5} = x^5 + 5 \) at the point \((1, \frac{6}{31})\).

7. (1 pt) 0.625 Find the coordinates of those points on the curve given by the equation \( x^2 - 1.25xy + y^2 = 16 \) at which the tangent line has slope 1. The first point must be the one with the greater \( x \) coordinate.

8. (1 pt) Find the \( x \) coordinate of the point on the curve \( x^2 + 4xy = 10 \) where the tangent line is parallel to the line \( 9x + y = 2 \). There are two answers; submit the greater answer first.

9. (1 pt) If the variables \( s \) and \( t \) are related by the equation \( st + 5t^3 = 9 \) find \( \frac{ds}{dt} \) implicitly.

10. (1 pt) If \( f \) is the focal length of a convex lens and an object is placed at a distance \( q \) from the lens, then its image will be at a distance \( p \) from the lens, where \( f \), \( q \), and \( p \) are related by the lens equation
   \( \frac{1}{f} = \frac{1}{q} + \frac{1}{p} \)

   Suppose the focal length of a particular lens is 20 cm. What is the rate of change of \( q \) with respect to \( p \) when \( p = 10 \)? (Make sure you have the correct sign for the rate.)

11. (1 pt) A street light is at the top of a 14.500 ft. tall pole. A man 5.800 ft tall walks away from the pole with a speed of 7.000 feet/sec along a straight path. How fast is the tip of his shadow moving away from the pole when the man is 39.000 feet from the pole?

12. (1 pt) A spherical snowball is melting in such a way that its radius is decreasing at the rate of 0.4 cm/min. At what rate is the volume of the snowball decreasing when the radius is 6.5 cm? (Note the answer is a positive number).

13. (1 pt) Sand falls out of the end of a slurry at the rate of 120 cc/sec. The pile forms a circular cone, the ratio of whose base diameter to height is 3. When the pile is of height 70 cm., at what rate is the height of the pile increasing?

14. (1 pt) Water is flowing into a balloon at the rate of 20 cc/min. The balloon has a puncture, and the rate at which water flows out of the balloon at
this puncture is proportional to the volume of water in the balloon. Let the proportionality constant be 0.24. At what volume do we have equilibrium (that is the volume of water in the balloon remains constant)? Hint: if we let $W_i$ represent the amount of water which flows in, and $W_o$ the amount flowing out of the puncture, we have $\frac{dW_i}{dt} = 20$ and $\frac{dW_o}{dt} = 0.24V$ where $V$ is the volume of water in the balloon.

$V = \underline{15}.$

(1 pt) Given that $\frac{d}{dx}(f(3x^3)) = 5x^4$, find a formula for the derivative of $f$.

Hint: let $u = 3x^3$, and write down the chain rule: $df/dx = (df/du)(du/dx)$, and solve for $df/du$. The answer is in terms of $x$; rewrite it in terms of $u$.

16. (1 pt) If the variables $x$ and $y$ are related by the equation

$$x^3 + y^3 = 2$$

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ implicitly.

(a) $\frac{dy}{dx} = \underline{\hspace{2cm}}$

(b) $\frac{d^2y}{dx^2} = \underline{\hspace{2cm}}$

(For part (b), differentiate part (a) implicitly, substitute for $dy/dx$ using part (a), and simplify using the original equation.)
Qualitative properties of functions and graphs.

This assignment will cover the material from Chapters 4.1 – 4.3.

1. (1 pt) Enter a T or an F in each answer space below to indicate whether the corresponding statement is true or false.

A good technique is to think of several examples, especially examples which might show that the statement is false!

You must get all of the answers correct to receive credit.

1. Every continuous function whose domain is a bounded, closed interval and which has a maximum value also has a minimum value.
2. Every differentiable function whose domain is a bounded, closed interval and which has a maximum value also has a minimum value.
3. If a function is increasing near a point \( a \) then its linear approximation at \( a \) cannot be decreasing.
4. If a continuous function has a maximum value then it also has a minimum value.
5. Every continuous function whose domain is a bounded, closed interval has a maximum value.
6. Every continuous function has a maximum value.
7. If a differentiable function has a maximum value then it also has a minimum value.
8. If a differentiable function \( f(x) \) has a maximum value on an interval then the function \( -f(x) \) has a minimum on that same interval.

2. (1 pt) What number exceeds its square by the maximum amount? Begin by convincing yourself that this number is on the interval \([0,1]\).

Answer: ____________

3. (1 pt) Consider the function \( f(x) = 3 - 4x^2 \), \(-5 \leq x \leq 1\).

The absolute maximum value is ____________
and this occurs at \( x \) equals ____________
The absolute minimum value is ____________

4. (1 pt) Let \( Q = (0,2) \) and \( R = (12,7) \) be given points in the plane. We want to find the point \( P = (x,0) \) on the \( x \)-axis such that the sum of distances \( PQ + PR \) is as small as possible. (Before proceeding with this problem, draw a picture!)

To solve this problem, we need to minimize the following function of \( x \):

\[ f(x) = \text{__________} \]

over the closed interval \([a,b]\) where \( a = \text{____} \) and \( b = \text{____} \).

5. (1 pt) The function

\[ f(x) = 2x^3 - 12x^2 - 126x + 0 \]

is decreasing on the interval (_____ , _____).

It is increasing on the interval (_____ , _____) and the interval (_____ , _____).

The function has a local maximum at _____.

6. (1 pt) For \( x \in [-13,13] \) the function \( f \) is defined by

\[ f(x) = x^4(x - 6)^3 \]

On which two intervals is the function increasing?
_____ to _____
and
_____ to ____

Find the region in which the function is positive:
_____ to _____

Where does the function achieve its minimum?

7. (1 pt) Identify the critical points and find the maximum value and minimum value of the following function on the given interval.

\[ f(x) = x^3 - 3x + 1, \text{ over } [-3/2,3]. \]

Critical Points: ____________

Maximum: ____________
Minimum: ____________

Instructions:
1) When entering the critical points, please enter them in the order that they appear on the real line.
2) If the function has no critical points, enter the string NONE in all answer boxes for critical points.

8. (1 pt) Consider the function \( f(x) = 6(x - 2)^{2/3} \).

For this function there are two important intervals: \((-\infty, A) \) and \((A, \infty) \) where \( A \) is a critical number.

Find \( A \) ____________
For each of the following intervals, tell whether $f(x)$ is increasing (type in INC) or decreasing (type in DEC).

$(-\infty, A): ____$

$(A, \infty): ____$

9. (1 pt) Find the length of the shortest line from the origin to the line $y = 1 - 1x$.

$d = ________$

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
WeBWorK Assignment 7 due 7/11/06 at 10:59 PM

Max-min applications, mean values, and sophisticated graphing.
This assignment will cover the material from Chapters 4.4 – 4.8.

1. (1 pt) A racer can cycle around a circular loop at the rate of 3 revolutions per hour. Another cyclist can cycle the same loop at the rate of 8 revolutions per hour. If they start at the same time (t=0), at what first time are they farthest apart?

\[ t = \ \text{hours}. \]

2. (1 pt) Two men are at opposite corners of a square block which is 500 feet on a side. They start to walk at the same time; one man walking east at the rate of 8 feet per second, and the other walks west at the rate of 4 feet per second. At what time are they closest?

\[ t = \ \text{seconds}. \]

3. (1 pt) A rectangle is to be drawn in the first quadrant with one leg on the y-axis, and the other on the x-axis, and a vertex on the curve \( y = 1 - 0.18x^2 \). Find the coordinates of that vertex which form the rectangle of greatest area.

\[ x = \quad y = \]

4. (1 pt) The illumination at a point is inversely proportional to the square of the distance of the point from the light source and directly proportional to the intensity of the light source. If two light sources are 30 feet apart and their intensities are 30 and 10 respectively, at what point between them will the sum of their illuminations be a minimum?

Solution:

Let \( x \) be the distance from the dimmer source at which the sum of the illuminations is a minimum. Then

\[ x = \ \text{feet}. \]

5. (1 pt) A rectangle with sides parallel to the coordinate axes is inscribed in the ellipse

\[ 25x^2 + 25y^2 = 625. \]

Find the dimensions of the rectangle of greatest area.

Answer:

\[ x = \quad y = \]

6. (1 pt) A drum in the form of a circular cylinder and open at one of the circular ends, is to be made so as to contain one cubic yard. Find the dimensions of the drum (height \( h \) and base radius \( r \)) which minimizes the amount of material going into the drum. The surface area of the drum includes the area of the cylinder and the circles at bottom.

\[ r = \quad h = \]

7. (1 pt) The Miraculous Widget Company is informed by its market analyst, that if the price of widgets is $50-x, the company will sell 100+8x thousand widgets. At what price should the widget be set so the company maximizes income?

\[ x = \quad \text{price} = \]

8. (1 pt) Another company sells wrought iron umbrella trees. This company has fixed costs of $100/month (in thousands of dollars). The cost in labor and material is $15 per umbrella. To sell \( x \) items, the price must be set at $50-0.8x, where \( x \) is in thousands of umbrellas sold per month. What price will maximize profit?

\[ \text{price} = \]

9. (1 pt) Consider the function \( f(x) = 5x + 8x^{-1} \). For this function there are four important intervals: \(( -\infty, A], [A, B),(B, C], \) and \([C, \infty) \) where \( A \), and \( C \) are the critical numbers and the function is not defined at \( B \).

Find \( A \) _____

and \( B \) _____

and \( C \) _____

For each of the following intervals, tell whether \( f(x) \) is increasing (type in INC) or decreasing (type in DEC).

\( (-\infty, A] \): _____

\( [A, B) \): _____

\( (B, C) \): _____

\( [C, \infty) \) _____

Note that this function has no inflection points, but we can still consider its concavity. For each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).

\( (-\infty, B) \): _____
10. (1 pt) Answer the following questions for the function 
\[ f(x) = x\sqrt{x^2 + 16} \]
defined on the interval \([-4, 7]\).

A. \( f(x) \) is concave down on the region _____ to _____
B. \( f(x) \) is concave up on the region _____ to _____
C. The inflection point for this function is at _____
D. The minimum for this function occurs at _____
E. The maximum for this function occurs at _____

11. (1 pt) Answer the following questions for the function 
\[ f(x) = x\sqrt{x^2 - 2x + 2} - 1\sqrt{x^2 - 2x + 2} \]
defined on the interval \([-5, 5]\). If you factor out the radical and then note the similarity to the preceding problem, it becomes easy.

A. \( f(x) \) is concave down on the region _____ to _____
B. \( f(x) \) is concave up on the region _____ to _____
C. The inflection point for this function is at _____
D. The minimum for this function occurs at _____
E. The maximum for this function occurs at _____

12. (1 pt) Consider the function \( f(x) = 12x^5 + 45x^4 - 200x^3 + 2 \).
\( f(x) \) has inflection points at (reading from left to right) \( x = D, E, F \) where \( D \) is _____
and \( E \) is _____
and \( F \) is _____
For each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).
\( (-\infty, D]: _____ \)
\( [D, E]: _____ \)
\( [E, F]: _____ \)

13. (1 pt) Consider the function \( f(x) = \frac{3x + 2}{6x + 2} \). For this function there are two important intervals: \(( -\infty, A) \) and \(( A, \infty) \) where the function is not defined at \( A \).

Find \( A _____ \)
For each of the following intervals, tell whether \( f(x) \) is increasing (type in INC) or decreasing (type in DEC).
\( (-\infty, A]: _____ \)
\( (A, \infty) _____ \)
Note that this function has no inflection points, but we can still consider its concavity. For each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).
\( (-\infty, A]: _____ \)
\( (A, \infty) _____ \)

14. (1 pt) Consider the function \( f(x) = -2x^3 + 24x^2 - 42x + 4 \). For this function there are three important intervals: \(( -\infty, A], [A, B], \) and \([B, \infty) \) where \( A \) and \( B \) are the critical numbers.

Find \( A _____ \) and \( B _____ \)
For each of the following intervals, tell whether \( f(x) \) is increasing (type in INC) or decreasing (type in DEC).
\( (-\infty, A]: _____ \)
\( [A, B]: _____ \)
\( [B, \infty) _____ \)
f(\( x \)) has an inflection point at \( x = C \) where \( C _____ \)
Finally for each of the following intervals, tell whether \( f(x) \) is concave up (type in CU) or concave down (type in CD).
\( (-\infty, C]: _____ \)
\( [C, \infty) _____ \)

15. (1 pt) NOTE: If there is no correct answer, submit NO.
Let 
\[ y = \frac{x^2}{10x + 8} \]
a. The graph has vertical asymptotes along the lines \( x = a \) for \( a = _____ \).
b. The horizontal asymptote is
c. As x approaches a from the left, y approaches 

d. As x approaches a from the right, y approaches 

e. The graph has a local maximum at 
x = 

f. The graph has a local minimum at 
x = 

g. The graph is increasing in the intervals (______  , ______) and (______  , ______).

16. (1 pt) NOTE: If there is no correct answer, submit NO.

Let 

\[ y = \frac{x + 3}{(x - 8)^2} \]

a. The graph has a vertical asymptote x=a for 
a =

b. The horizontal asymptote is 
y =

c. As x approaches a from the left, y approaches 

d. As x approaches a from the right, y approaches 

e. The graph has a local maximum at 
x =

f. The graph has a local minimum at 
x =

g. The graph is increasing in the intervals (______  , ______) and (______  , ______).

17. (1 pt) NOTE: If there is no correct answer, submit NO.

Let 

\[ y = \sqrt{8x^2 + 10} \]

a. The horizontal asymptote is 
y =

b. The graph has a local minimum at 
x =

c. The minimum value of y is 

18. (1 pt) NOTE: If there is no correct answer, submit NO.

Consider the function

\[ y = \frac{\sin x}{2 + \cos x} \]

defined over the interval \([-\pi/2, \pi/2]\).

a. The graph has a vertical asymptote at 
x =

b. \( y < 0 \) for x in the interval between 

and 

c. \( y \) is increasing for x in the interval between 

and 

19. (1 pt) A rancher wants to fence in an area of 3000000 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use?

Note: the answer is to be the length of fence which is a minimum, not the length of a side which achieves that minimum.

20. (1 pt) Consider the function

\[ f(x) = -2x^3 + 2x^2 + 3x - 2 \]

Find the average slope of this function on the interval \((2, 7)\).

By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((2, 7)\) such that \( f'(c) \) is equal to this mean slope. Find the value of \( c \) in the interval which works.

21. (1 pt) Consider the function \( f(x) = \frac{1}{x} \) on the interval \([5, 8]\). Find the average or mean slope of the function on this interval.

By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((5, 8)\) such that \( f'(c) \) is equal to this mean slope. For this problem, there is only one \( c \) that works. Find it.
Antiderivatives and Differential Equations.
This assignment will cover the material from Chapters 5.1 – 5.2.

1. (1 pt) Consider the function \( f(t) = 7\sec^2(t) - 9t^3 \). Let \( F(t) \) be the antiderivative of \( f(t) \) with \( F(0) = 0 \).
Then \( F(t) \) equals 

2. (1 pt) Evaluate the integral: \( \int \frac{s(s+1)^2}{\sqrt{s}} ds \).
Answer: \( \quad \) + C.

3. (1 pt) Evaluate the indefinite integral:
\[ \int \frac{3y}{\sqrt{2y^2 + 5}} dy. \]
Answer: \( \quad \) + C.

4. (1 pt) Consider the differential equation:
\[ \frac{du}{dt} = -u^2(t^3 - t). \]
a) Find the general solution to the above differential equation. (Instruction: Write the answer in a form such that its numerator is 1 and its integration constant is \( C \) — rename your constant if necessary.)
Answer: \( u = \quad \)
b) Find the particular solution of the above differential equation that satisfies the condition \( u = 4 \) at \( t = 0 \).
Answer: \( u = \quad \)

5. (1 pt) Given \( f''(x) = -16\sin(4x) \) and \( f'(0) = -5 \) and \( f(0) = 3 \).
Find \( f(\pi/2) = \quad \)

6. (1 pt) Given
\[ f''(x) = 4x + 0 \]
and \( f'(-1) = 3 \) and \( f(-1) = 6 \).
Find \( f'(x) = \quad \)
and find \( f(3) = \quad \)

7. (1 pt) Find
\[ F(x) = \int x(x^2 + 3)^6 dx \]
Give a specific function for \( F(x) \).
\( F(x) = \quad \)

8. (1 pt) Evaluate the indefinite integral.
\[ \int x^2 \sqrt{1 + x^3} dx \]

9. (1 pt) Find a function \( y \) of \( x \) such that
\( 6yy' = x \) and \( y(6) = 2 \).
\( y = \quad \) (function of \( x \))

10. (1 pt) Solve the separable differential equation
\[ 4x - 2y\sqrt{x^2 + 1} \frac{dy}{dx} = 0 \]
Subject to the initial condition: \( y(0) = 8 \)
\( y = \quad \) (function of \( x \) only)

Prepared by the WeBWorK group, Dept. of Mathematics, University of Rochester, ©UR
1. (1 pt) If \( f(x) = \int_4^x t^5 \, dt \)
then
\[
f'(x) = \frac{5}{6} x^{5/6}
\]
\[
f'(2) = \frac{5}{6} 2^{5/6}
\]

2. (1 pt)
\[
\int_2^{19} f(x) - \int_2^{11} f(x) = \int_a^b f(x)
\]
where \( a = \) and \( b = \) 19

3. (1 pt) Let \( \int_{-10}^{-4} f(x) \, dx = 4 \), \( \int_{-10}^{-8} f(x) \, dx = 10 \), \( \int_{-6}^{-4} f(x) \, dx = 2 \).
Find \( \int_{-8}^{-6} f(x) \, dx = \) and \( \int_{-6}^{-8} (4f(x) - 10) \, dx = \)

4. (1 pt) Consider the function \( f(x) = \frac{x^2}{4} - 7 \).
In this problem you will calculate \( \int_0^3 \left( \frac{x^2}{3} - 7 \right) \, dx \)
by using the definition
\[
\int_a^b f(x) \, dx = \lim_{n \to \infty} \left[ \sum_{i=1}^n f(x_i) \Delta x \right]
\]
The summation inside the brackets is \( R_n \) which is the Riemann sum where the sample points are chosen to be the right-hand endpoints of each sub-interval.
Calculate \( R_n \) for \( f(x) = \frac{x^2}{4} - 7 \) on the interval \([0, 3]\) and write your answer as a function of \( n \) without any summation signs. You will need the summation formulas on page 381 of your textbook (page 364 in older texts).
\[
R_n = \frac{1}{4} \sum_{i=1}^n (i-1)^2
\]
\[
\lim_{n \to \infty} R_n = \frac{1}{12}
\]

5. (1 pt) The following sum
\[
\frac{1}{1 + \frac{3}{n}} + \frac{1}{1 + \frac{6}{n}} + \frac{1}{1 + \frac{9}{n}} + \ldots + \frac{1}{1 + \frac{3n}{n}}
\]
is a right Riemann sum for a certain definite integral
\[
\int_1^b f(x) \, dx
\]
using a partition of the interval \([1, b]\) into \( n \) subintervals of equal length.
Then the upper limit of integration must be: \( b = \)
and the integrand must be the function \( f(x) = \)

6. (1 pt) If \( f(x) = \int_x^0 t^4 \, dt \)
then
\[
f'(x) = -x^3
\]

7. (1 pt) If \( f(x) = \int_0^x (t^3 + 7t^2 + 1) \, dt \)
then
\[
f''(x) = 3x^2 + 14x + 1
\]

8. (1 pt) Given
\[
f(x) = \int_0^x \frac{t^2 - 25}{1 + \cos^2(t)} \, dt
\]
At what value of \( x \) does the local max of \( f(x) \) occur?
\[
x = \pi
\]

9. (1 pt) NOTE: It will be easier to see the function \( f(x) \) if you use the display mode “typeset”. Keep in mind, though, that loading the problem into your computer using this display mode will take longer.
Let
\[
f(x) = \begin{cases} 
0 & \text{if } x < -5 \\
3 & \text{if } -5 \leq x < 0 \\
-5 & \text{if } 0 \leq x < 4 \\
0 & \text{if } x \geq 4
\end{cases}
\]
and
\[
g(x) = \int_{-5}^x f(t) \, dt
\]
Determine the value of each of the following:
(a) \( g(-8) = \)
(b) \( g(-4) = \)
(c) \( g(1) = \)
(d) \( g(5) = \)
The absolute maximum of \( g(x) \) occurs when \( x = \) ___ and is the value ___.

It may be helpful to make a graph of \( f(x) \) when answering these questions.

10. (1 pt) Use part I of the Fundamental Theorem of Calculus to find the derivative of

\[
F(x) = \int_x^6 \sin(t^4) \, dt
\]

\[ F'(x) = \text{__________} \]

[NOTE: Enter a function as your answer.]

11. (1 pt) Use part I of the Fundamental Theorem of Calculus to find the derivative of

\[
h(x) = \int_{-5}^{\sin(x)} (\cos(t^4) + t) \, dt
\]

\[ h'(x) = \text{__________} \]

[NOTE: Enter a function as your answer. Make sure that your syntax is correct, i.e. remember to put all the necessary *, (, ), etc.]

12. (1 pt) Find the derivative of the following function

\[
F(x) = \int_x^3 (2t - 1)^3 \, dt
\]

using the Fundamental Theorem of Calculus.

\[ F'(x) = \text{__________} \]

13. (1 pt) Find a function \( f \) and a number \( a \) such that

\[
2 + \int_a^x \frac{f(t)}{t^4} \, dt = 4x^{-1}
\]

\[ f(x) = \text{__________} \]

\[ a = \text{__________} \]

14. (1 pt) Evaluate the definite integral

\[
\int_3^4 \left( \frac{d}{dt} \sqrt{5 + 4t^4} \right) \, dt
\]

using the Fundamental Theorem of Calculus.

You will need accuracy to at least 4 decimal places for your numerical answer to be accepted. You can also leave your answer as an algebraic expression involving square roots.

\[
\int_3^4 \left( \frac{d}{dt} \sqrt{5 + 4t^4} \right) \, dt = \text{__________}
\]
Integral MVT, Area Between Curves, and Volume of Revolution.

This assignment will cover the material from Chapters 5.7 – 6.2.

1. (1 pt) Consider the function \( f(x) = x^2 - x \) on the interval \([-6, 9]\). Find the average or mean slope of the function on this interval.

By the Mean Value Theorem, we know there exists a \( c \) in the open interval \((-6, 9)\) such that \( f'(c) \) is equal to this mean slope. Find \( c \).

\[ c = \text{_____} \]

2. (1 pt) Find the volume of the solid formed by rotating the region inside the first quadrant enclosed by 
\[ y = x^3 \]
\[ y = 25x \]
about the x-axis.

3. (1 pt) A ball of radius 10 has a round hole of radius 4 drilled through its center. Find the volume of the resulting solid.

4. (1 pt) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = 9x^2, x = 1, y = 0, \text{ about the x-axis} \]

5. (1 pt) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.
\[ y = 0, y = x(4 - x) \text{ about the axis } x = 0 \]

6. (1 pt) The value of \( \int_{0}^{-7} (x - 2)^2 \, dx \) is

7. (1 pt) The value of \( \int_{3}^{4} \frac{1}{x^2} \, dx \) is

8. (1 pt) Evaluate the definite integral
\[ \int_{4}^{7} \frac{4x^2 + 7}{\sqrt{x}} \, dx \]

9. (1 pt) Evaluate
\[ I = \int_{0}^{8} x^2 \sqrt{x^3 + 7} \, dx \]

10. (1 pt) Evaluate
\[ I = \int_{5}^{7} \frac{x}{(x^2 + 1)^2} \, dx \]

11. (1 pt) Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.
\[ y = 4x, y = 4x^2 \]

12. (1 pt) Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.
\[ y = 7x^2, y = x^2 + 2 \]

13. (1 pt) Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.
\[ x + y^2 = 42, x + y = 0 \]

14. (1 pt) Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.
\[ y = 3\cos x, y = (4 \sec x)^2, x = -\pi/4, x = \pi/4 \]
Peter Alfeld  
Math 1210-90, Summer 2006  
WeBWorK Assignment 11 due 8/2/06 at 10:59 PM  
Volume of Revolution, Arclength, Moments, and Work  

This assignment will cover the material from Chapters 6.2 – 6.7.

1. (1 pt) Find the volume of the solid generated by revolving about the $x$-axis the region bounded by the upper half of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

and the $x$-axis, and thus find the volume of a prolate spheroid. Here $a$ and $b$ are positive constants, with $a > b$.

Volume of the solid of revolution: ________

2. (1 pt) The region bounded by $y = 2 + \sin x, y = 0, x = 0$ and $2\pi$ is revolved about the $y$-axis. Find the volume that results.

Hint:

$$\int x \sin x \, dx = \sin x - x \cos x + C.$$  

Volume of the solid of revolution: ________

3. (1 pt) Consider the parametric curve given by the equations

$$x(t) = t^2 + 32t - 25$$
$$y(t) = t^2 + 32t - 16$$

How many units of distance are covered by the point $P(t) = (x(t), y(t))$ between $t=0$, and $t=7$?

4. (1 pt) Find the area of the surface generated by revolving the following curve about the $x$-axis:

$$x = r \cos \theta, y = r \sin \theta, 0 \leq \theta \leq \pi,$$

where $r$ is a constant.

Area of the surface: ________

5. (1 pt) The circle $x = a \cos \theta, y = a \sin \theta, 0 \leq \theta \leq 2\pi$ is revolved about the line $x = b, 0 < a < b$, thus generating a torus (doughnut). Find its surface area.

Area of the torus: ________

6. (1 pt) Consider the parametric equation

$$x = 3(\cos \theta + \theta \sin \theta)$$

$$y = 3(\sin \theta - \theta \cos \theta)$$

What is the length of the curve for $\theta = 0$ to $\theta = \frac{\pi}{2}$?

7. (1 pt) If $f(\theta)$ is given by: $f(\theta) = 20\cos^3 \theta$ and $g(\theta)$ is given by: $g(\theta) = 20\sin^3 \theta$

Find the total length of the astroid described by $f(\theta)$ and $g(\theta)$.

(The astroid is the curve swept out by $(f(\theta), g(\theta))$ as $\theta$ ranges from 0 to $2\pi$.)

8. (1 pt) The force on a particle is described by $2x^3 - 1$ at a point $x$ along the $x$-axis. Find the work done in moving the particle from the origin to $x = 9$.

9. (1 pt) A force of 5 pounds is required to hold a spring stretched 0.1 feet beyond its natural length. How much work (in foot-pounds) is done in stretching the spring from its natural length to 0.8 feet beyond its natural length? ________

10. (1 pt) For a certain type of nonlinear spring, the force required to keep it stretched 8 inches is 2 pounds, how much work is done in stretching this spring 27 inches?

Amount of work done: ________ inch-pound(s).

11. (1 pt) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

$$y = x^2, y = 0, x = 0, x = 3$$

about the $y$-axis

12. (1 pt) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

$$y = \frac{1}{x}, y = 0, x = 2, x = 8$$

about the $y$-axis

13. (1 pt) A tank in the shape of an inverted right circular cone has height 9 meters and radius 11 meters. It is filled with 8 meters of hot chocolate.

Find the work required to empty the tank by pumping the hot chocolate over the top of the tank. Note: the density of hot chocolate is $\delta = 1530 \text{kg/m}^3$
14. (1 pt) Find the centroid of the region bounded by the following curves:

\[ y = x^2, \quad y = x + 3. \]

Hint: Make a sketch and use symmetry where possible.

Centroid: (________, ________).

15. (1 pt) A cylinder of radius 14 and depth 17 is filled with silver ore. The cost of extracting the ore lying at a depth of \( h \) feet is \( 120(1 + .03h^2) \) dollars per cubic foot. How much will it cost to empty the entire cylinder?

16. (1 pt) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = x^2, \quad y = 0, \quad x = 0, \quad x = 3, \] about the \( y \)-axis

17. (1 pt) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = \frac{1}{x}, \quad y = 0, \quad x = 4, \quad x = 7; \]
about the \( y \)-axis

18. (1 pt)

The base of a certain solid is the area bounded above by the graph of \( y = f(x) = 25 \) and below by the graph of \( y = g(x) = 36x^2 \). Cross-sections perpendicular to the \( x \)-axis are squares. (See picture above, click for a better view.)

Use the formula

\[ V = \int_a^b A(x) \, dx \]

to find the volume of the solid.

Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

The lower limit of integration is \( a = \) ________

The upper limit of integration is \( b = \) ________

The side \( s \) of the square cross-section is the following function of \( x \): \( A(x) = \) ________

Thus the volume of the solid is \( V = \) ________

19. (1 pt)

The base of a certain solid is an equilateral triangle with altitude 5. Cross-sections perpendicular to the altitude are semicircles. Find the volume of the solid, using the formula

\[ V = \int_a^b A(x) \, dx \]

applied to the picture shown above (click for a better view), with the left vertex of the triangle at the origin and the given altitude along the \( x \)-axis.

The volume of the solid is \( V = \) ________