Calculus III Practice Problems 9

1. Find the surface area of the part of the hyperbolic paraboloid z = xy that lies inside the cylinder $x^2 + y^2 \le 4$.

2. Find the surface area of the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

3. Find the surface area of the part of the surface $z = (2/3)(x^{3/2} + y^{3/2})$ that lies above the triangle in the first quadrant bounded by the line x + y = 1.

4. Find the mass and the *x*-coordinate of the center of mass of the solid bounded by the planes x = 0, y = 0, z = 0, x + y + z = 1 with the density function $\rho(x, y, z) = y$.

5. Find the center of mass of the piece of the solid parabolic shell $z \le 16 - (x^2 + y^2)$ lying above the xy-plane.

6. Find the average value of f(x, y, z) = x + y + z over the region *R* in the first octant (the region where all the coordinates are positive) under the plane x + y + z = 1.

7. The curve $z = (x-1)^2, 0 \le z \le 1$ is rotated about the *z*-axis, enclosing, together with the *xy*-plane, a 3-dimensional region *R*. *R* is filled with a substance whose density is inversely proportional to the distance from the *z*-axis. Find the total mass of this object.

8. Evaluate

$$\int \int \int_{R} (x^2 + y^2 + z^2) dx dy dz$$

where *R* is the ball $x^2 + y^2 + z^2 \le 4$.

9. Find the centroid of the region *R* described in example 24, Chapter 17.