Calculus III
Practice Problems 8

1. What is the mass of the lamina bounded by the curves $y = 3x$ and $y = 6x - x^2$ where the density function is $\delta(x, y) = xy$?

2. A lamina filled with a homogeneous material (the density is identicaly equal to 1) is in the shape of the region $0 \le x \le \pi$, $0 \le y \le \sin x$. Find its center of mass.

3. The surface H, given in cylindrical coordinates by $z2\theta$ is a helicoid. What is the volume of the region R bounded above by $H, 0 \le \theta \le 2\pi$, below by the plane $z = 0$ and lying over the disc $r \le 1$?

4. A beach B is shaped in the form of a crescent. We model this on the area between the circle of radius 1, centered at the origin, and the circle of radius 3/4 centered at the point (3/4,0), where the units are in miles. Suppose that the human density σ decreases as we move from the beach according to $\sigma(x, y) = 1000(x^2 + y^2)^{-2}$ people per square mile.What is the population on that beach?

5. The curve $z = (x - 1)^2, 0 \le z \le 1$ is rotated about the z-axis, enclosing, together with the xy-plane, a 3-dimensional region R. R is filled with a substance whose density is inversely proportional to the distance from the z-axis. Find the total mass of this object.

6. As (u, v) runs through the region $u^2 + v^2 \le 1$, the vector function

$$\mathbf{X}(u, v) = (u^2 + v^2)\mathbf{I} + (u^2 - v^2)\mathbf{J} + uv\mathbf{K}$$

describes a surface S in three space. Write down the double integral which must be calculated to find the surface area of S.

7. Find the volume of the region lying above the disc $x^2 + y^2 \le 1$ in the xy-plane, and below the surface $z = \sin(\pi\sqrt{x^2 + y^2}/2)$.

8. Find the mass of the lamina of the region R lying between the ellipses $x^2 + 4y^2 = 1$ and $x^2 + 4y^2 = 4$, where the density function is $\delta(x, y) = x^2 + y^2$.

9. Find the area of the region in the first quadrant bounded by the curves $y^2 = 2x$, $y^2 = 5x$, $x^2 = 4y$, $x^2 = 10y$.