Calculus III
Practice Problems 7

1. What is the volume of the region under the surface $z = \ln x + y$ and over the rectangle in the xy-plane with vertices (1,0), (1,5), (3,0), (3,5)?

2. What is the volume of the solid bounded by the surfaces $z = x^3$ and $z = x^2 + 2y^2$ lying directly over the rectangle $0 \le x \le 1,\ 0 \le y \le 3$?

3. What is the volume of the solid bounded above by the surface $z = y^2 - x^2$ lying directly over the triangle $T:\ 0 \le y \le 2, -y \le x \le y$?

4. What is the volume of the region under the curve $z = e^{x+y}$ and over the triangle in the xy-plane with vertices (0,0), (1,0), (0,2).

5. Let R be the region in the plane bounded by the curves $x = y^2$, $x = 3 - 2y^2$. Calculate

$$I = \int \int_R (y^2 - x)dxdy \ .$$

6. What is the mass of the lamina bounded by the curves $y = 1 + x$ and $y = 1 - x^3$ and the x-axis, where the density function is $\delta(x,y) = x^2$?

7. A lamina filled with a homogeneous material (the density is identicaly equal to 1) is in the shape of the region R bounded by the curves $y = 1$ and $y = x^2$. What is its center of mass?

8. Find the mass of the solid bounded by the surface $z = \sqrt{x^2 + y}$, the coordinate planes and the planes $x = 1$, $y = 2$, where the density is $\delta(x,y,z) = x$.