Calculus III
Practice Problems 6

1. Find the equation of the tangent plane to the surface given parametrically by

$$\mathbf{X}(u,v) = u^3\mathbf{I} + 2uv\mathbf{J} + v^2\mathbf{K}$$

at the point where $u = 1,\ v = 2$.

2. Let S be a surface which goes through the origin, and whose normal is the z-axis. Let Π be a plane containing the z-axis, and γ the curve of intersection of the surface S and the plane Π. Show that the principal normal to γ is $\pm\mathbf{K}$.

3. Let $f(x,y,z) = xyz - x^3 + x^2 + yz$. Find the critical points of f.

4. Let

$$f(x,y) = x^3 - 4y^3 + 3x^2y - 18x + 6.$$

Find all critical points and classify as maxima, minima, saddle points.

5. Let

$$g(x,y,z) = x^2y^2z.$$

Find the point on the surface $g(x,y,z) = 1$ which is closest to the origin.

6. (This is a Calculus I problem. You are to do it using the methods of Lagrange multipliers). John and Mary work part-time at the Widget factory, and are willing to work as much as 40 hours a week. John gets paid \$27/hour, and Mary gets \$45/hour. If John works x hours and Mary works y hours, they produce $3xy + (1/2)y^2$ widgets. The company has allocated \$1600/week for compensation to John and Mary (together). How many hours should they each work in order to produce the maximum number of widgets?

7. The material for the bottom of a box costs three times as much per square foot as the material for the sides and top. We wish to know the greatest volume such a box can have if the total maount of money available for material is \$12, and the material for the bottom costs \$0.60 per square foot. Find the system of equations which must be solved to get the answer.

8. Let $f(x,y) = x^2 + 2y^2 + 2x$. Find the minimum and maximum of f on the ball $x^2 + y^2 \le 16$.