Calculus III
Practice Problems 5

1. Let $f(x,y,z) = x\ln z + 2yz$. a) What is ∇f? b) Show that

$$\frac{\partial^2 f}{\partial z \partial x} = \frac{\partial^2 f}{\partial x \partial z}\,.$$

2. Find the direction **U** of maximal change for the function $w = x^3y^2z + xyz^2$ at the point (2,-1,2). What is $D_{\mathbf{U}}w$ at this point?

3. Suppose that the function $w = f(x,y)$ is differentiable at the point P in the plane. Let $\mathbf{V} = \mathbf{I} + 2\mathbf{J}$, $\mathbf{W} = \mathbf{I} - \mathbf{J}$, and that $D_{\mathbf{V}}w = 2$, $D_{\mathbf{W}}w = 3$ at P. What is $\nabla w(P)$?

4. Suppose $z = f(x,y)$ is differentiable at (1,1), and suppose that

$$\frac{d}{dt} f(1+t, 1+t^2)|_{t=0} = 3, \quad \frac{d}{dt} f(1, 1+t)|_{t=0} = 2\,.$$

What is $\nabla f(1,1)$?

5. A particle moves in space according to the equation

$$\mathbf{X}(t) = t^2\mathbf{I} + (1-t^2)\mathbf{J} + (1-t)\mathbf{K}\,.$$

For $w = xy + yz^2 + xz^2$, find dw/dt along the trajectory. What is dw/dt when $t = 2$?

6. Let $w = xyz$ and let γ be the helix given by

$$\mathbf{X}(t) = \cos t\mathbf{I} + \sin t\mathbf{J} + t\mathbf{k}\,.$$

Find dw/dt at $t = 2\pi/3$.

7. Find the equation of the tangent plane to the surface

$$x^{1/2} + y^{1/2} - z^{1/2} = 0$$

at the point (4,9,25).

8. Find the equation of the tangent plane to the surface

$$x\ln z + 2yz = 0$$

at the point $(-e^2, 1, e^2)$.