

Calculus III
Practice Problems 4

1. Consider the line L in the plane given by the equation $2x + 5y + 10 = 0$. Find a base $\{\mathbf{U}, \mathbf{V}\}$ with $\mathbf{U}$ parallel to L , and $\mathbf{V}$ counterclockwise to $\mathbf{U}$. Find the equation of the line in coordinates $\{u, v\}$ relative to the base $\{\mathbf{U}, \mathbf{V}\}$.
2. An ellipse has center at the point $(2, 1)$, and its major axis is the line $x + y = 3$. Its major radius is 3 and its minor radius is 1. What is the equation of the ellipse?
3. Show that the intersection of a plane with a sphere is a circle.
4. Consider the set of all points P in space such that the vector from O to P has length 2 and makes an angle of 45 degrees with $\mathbf{I} + \mathbf{J}$.
 - a) What kind of geometric object is this set?
 - b) Give equations in cartesian coordinates for this set.

5. Write down the equations of the paraboloid of revolution $z = x^2 + y^2$ in cylindrical and spherical coordinates.

6. a) Draw some typical level curves in the (x, y) -plane for the function

$$f(x, y) = (1 + x^2 + y^2)^{-1}.$$

b) Sketch the surface $z = f(x, y)$.

7. Let L be the line $x = 1, z = 3y$. If we rotate the line about the z -axis, it describes a surface. Find the equation of that surface.

8. Consider the helix $\mathbf{X}(t) = \cos t \mathbf{I} + \sin t \mathbf{J} + t \mathbf{K}$. For each t , let L_t be the line perpendicular to the z -axis intersecting the helix at $\mathbf{X}(t)$. Find the equation of the surface swept out by the lines L_t . What are its level sets?

9. Sketch or describe the surface given by the equation

$$\frac{y^2}{4} - \frac{z^2}{9} = x^2.$$

10. Sketch or describe the surface given by the equation

$$\frac{x^2}{9} + \frac{z^2}{4} = y.$$