Calculus III
Practice Problems 3

1. Let $\mathbf{L} = \cos\theta(t)\mathbf{I} + \sin\theta(t)\mathbf{J}$ be a unit vector -valued function of t in the xy-plane. Show that

$$\frac{d}{dt}(\mathbf{L}\times\mathbf{K}) = \frac{d\theta}{dt}\mathbf{L}.$$

2. A particle moves in the plane according to the equation

$$\mathbf{X}(t) = t\sin t\mathbf{I} + \cos t\mathbf{J}$$

Find the speed, tangential and normal accelerations and the curvature of the trajectory at the time $t = 2\pi$.

3. A particle moves in the plane according to the equation

$$\mathbf{X}(t) = (t^2 + t + 1)\mathbf{I} + t^3\mathbf{J}$$

Find the velocity, speed, acceleration, tangent and normal vectors of the particle at any time $t > 0$.

4. A particle moves in the plane according to the equation

$$\mathbf{X}(t) = t\mathbf{I} - \ln t\mathbf{J}$$

Find the velocity, speed, acceleration, tangent and normal vectors, and tangential and normal acceleration of the particle at any time $t > 0$.

5. A particle moves in the plane according to the equation

$$\mathbf{X}(t) = e^{at}(\cos t\mathbf{I} + \sin t\mathbf{J})$$

Show that the angle between the position vector and the tangent line to the trajectory is constant.

6. A particle moves in space according to the formula

$$\mathbf{X}(t) = \cos t\mathbf{I} + \sin t\mathbf{J} + \cos(2t)\mathbf{K}\ .$$

Find the tangential and normal accelerations and the curvature at $t = \pi/4$.

7. A particle moves in space according to the formula

$$\mathbf{X}(t) = \cos t\mathbf{I} + \sin t\mathbf{J}) + e^t\mathbf{K}\ .$$

Find the tangential and normal accelerations for this motion.

8. A particle moves in space according to the formula

$$\mathbf{X}(t) = \frac{1}{2}t^2\mathbf{I} + \frac{1}{t}\mathbf{J} - t\mathbf{K}\ .$$

Find a_N, κ at the point $t = 1$.

9. Let $\mathbf{U}(t),\ \mathbf{V}(t)$ be unit vectors which are always orthogonal. Let $\mathbf{W} = \mathbf{U}\times\mathbf{V}$. Show that there are scalar functions $\alpha,\ \beta,\ \gamma$ such that

$$\frac{d\mathbf{U}}{dt} = \alpha\mathbf{V} + \beta\mathbf{W}\ , \quad \frac{d\mathbf{V}}{dt} = -\alpha\mathbf{U} + \gamma\mathbf{W}\ , \quad \frac{d\mathbf{W}}{dt} = -\beta\mathbf{U} - \gamma\mathbf{V}$$

10. Let $\mathbf{X} = \mathbf{X}(t)$ represent the motion of a particle in space.
a) Show that if $|\mathbf{V}|$ is constant, then $\mathbf{A}$ is orthogonal to $\mathbf{T}$.
b) Show that if $\mathbf{A}$ is constant, the motion lies in a plane.