Calculus III
Practice Problems 2

1. Find a unit vector orthogonal to the vectors $\mathbf{V} = 6\mathbf{I} - 7\mathbf{J} + \mathbf{K},\ \mathbf{W} = -\mathbf{I} + 2\mathbf{J} - 3\mathbf{K}$. What is the volume of the parallelopiped determined by these three vectors?

2. Find a vector which makes an angle of $30°$ with the plane determined by the vectors $\mathbf{V} = 6\mathbf{I} - 7\mathbf{J} + \mathbf{K},\ \mathbf{W} = -\mathbf{I} + 2\mathbf{J} - 3\mathbf{K}$.

3. Find a vector normal to the plane given by the equation

$$3x - 2y + z = 14\ .$$

4. Find a vector normal to the plane through $P : (0,0,0),\ Q : (1,0,-1),\ R : (0,1,1)$.

5. Find the equation of the plane through the origin which is normal to the line given parametrically by

$$\mathbf{X} = (3\mathbf{I} + 2\mathbf{J} - \mathbf{K}) + t(-\mathbf{I} + \mathbf{J} + 2\mathbf{K})\ .$$

6. Find $\mathbf{V_1} \cdot (\mathbf{V_2} \times \mathbf{V_3})$ where

$$\mathbf{V_1} = -\mathbf{I} + 2\mathbf{J} + \mathbf{K}. \quad \mathbf{V_2} = 2\mathbf{I} - 2\mathbf{J} + 3\mathbf{K}, \quad \mathbf{V_3} = \mathbf{I} - 2\mathbf{K}\ .$$

7. Show that if $\det(\mathbf{U},\mathbf{V},\mathbf{W}) = |\mathbf{U}||\mathbf{V}||\mathbf{W}|$, then the three vectors are mutually orthogonal.

8. Find the symmetric equations of the line through the point (2,-1,3) which is perpendicular to the vectors $\mathbf{V} = 2\mathbf{I} - \mathbf{J} + 3\mathbf{K}$ and $\mathbf{W} = \mathbf{I} - \mathbf{J} + \mathbf{K}$.

9. Find the parametric equations of the line through the point (2,-1,3) which is parallel to the two planes given by the equations

$$3x + z = 4, \qquad x - 2y + 5z = 1\ .$$

10. Find the equation of the plane through the point (2,-1,3) which is parallel to the vectors $\mathbf{I} - 2\mathbf{J} + 3\mathbf{K}$ and $3\mathbf{I} - 2\mathbf{J} + \mathbf{K}$.

11. Find the distance of the point $(2,0,1)$ from the plane given by the equation

$$\frac{x-2}{3} + \frac{y+1}{4} + \frac{z-1}{2} = 0$$

12. Let $L_1,\ L_2$ be two lines in three dimensions which do not intersect. There are points $P_1,\ P_2$ on $L_1,\ L_2$ respectively such that the line joining P_1 and P_2 intersects each line at a right angle. These are the points on the two lines which are closest together. Find a formula for the distance between P_1 and P_2, in terms of the equations of the lines

$$L_1 : \quad \mathbf{X} = \mathbf{Q_1} + t\mathbf{N_1} \qquad L_2 : \quad \mathbf{X} = \mathbf{Q_2} + s\mathbf{N_2}\ .$$