Calculus III
Practice Problems 11

1. Let C be the boundary of the triangle with vertices at (0,0), (3,0), (4,5), oriented in the counterclockwise sense. Find $\int_C 3y\,dx + 6x\,dy$.

2. Find $\int_C xy^2dx + x^2ydy$ where C is the line from (2,3) to (5,1).

3. Let C be the boundary of the square $0 \le x \le \pi,\ 0 \le x \le \pi$, traversed in the counterclockwise sense. Find

$$\int_C \sin(x+y)dx + \cos(x+y)dy\ .$$

4. Let C be the boundary of the triangle with vertices (0,0), (1,0), (1,2), traversed in the counterclockwise sense. Find $\int_C x^2dx + xydy$.

5. The **cycloid** is the curve given parametrically in the plane by

$$x = t - \sin t\ , \quad y = 1 - \cos t\ , \quad t \ge 0\ .$$

Find the area under one arch of the cycloid.

6. A function of two variables is **harmonic** if

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0\ .$$

Let R be a regular region in the plane with boundary C, $\mathbf{N}$ the exterior normal to C. Show that

$$\oint_C \nabla f \cdot \mathbf{N} ds = 0\ .$$

7. Let $\mathbf{F}(\mathbf{X}) = e^{xy}(\mathbf{I}+\mathbf{J})$, and let R be the rectangle $0 \le x \le 2,\ -1 \le y \le 1$. For C the boundary of R traversed counterclockwise, Calculate

$$\text{a)} \quad \oint_C \mathbf{F}\cdot\mathbf{T}ds \qquad \text{b)} \quad \oint_C \mathbf{F}\cdot\mathbf{N}ds\ ,$$

where $\mathbf{T}$ and $\mathbf{N}$ are the tangent and normal to the curve C.

8. Let $\mathbf{F} = x\mathbf{I} + xy^2\mathbf{J}$. Let C be the circle $x^2+y^2 = 1$ traversed in the counterclockwise sense. Find

$$\int_C \mathbf{F}\cdot\mathbf{T}ds\ , \qquad \int_C \mathbf{F}\cdot\mathbf{N}ds$$

where s represents arclength along the circle, $\mathbf{T}$ is the unit tangent vector, and $\mathbf{N}$ is the unit external normal vector.

9. Let $\mathbf{F} = y\mathbf{I} + 2x\mathbf{J}$. Let C be the curve $y = x^2$ from $x = 0$ to $x = 2$. Find the flux of $\mathbf{F}$ across C from left to right, that is, for $\mathbf{N}$ the unit normal to the right along C, find

$$\int_C \mathbf{F}\cdot\mathbf{N}ds\ .$$

10. Let $\mathbf{F} = x^3\mathbf{I} + y^3\mathbf{J}$. Let C be the circle $x^2+y^2 = 9$ traversed in the counterclockwise sense. Find

$$\int_C \mathbf{F}\cdot\mathbf{N}ds\ .$$

where s represents arclength along the circle, and $\mathbf{N}$ is the unit external normal vector.