

Calculus III
Practice Problems 1

1. Find the components of a vector $\mathbf{V}$ making an angle of 45° with $\mathbf{W} = 2\mathbf{I} - \mathbf{J}$.
2. Show that the diagonals of a rhombus intersect at right angles. (A *rhombus* is a parallelogram with all sides of the same length.)
3. A river ferry runs at a speed of 6 knots across a river with a current of 2 knots. Assume that the river has shores which are parallel straight lines. In what direction should the barge head in order to cross the river perpendicular to the shores?
4. Find the area of the parallelogram determined by the vectors $\mathbf{V} = 6\mathbf{I} - 7\mathbf{J}$, $\mathbf{W} = 3\mathbf{I} + 4\mathbf{J}$. What are the coordinates of the vertex of this parallelogram farthest from the origin?
5. A plane flies at a ground speed of 480 mph. The jet stream comes from 30° north of west at 60 mph. In what direction should the pilot direct the plane so as to be heading directly east? How many miles will the plane cover in 2 hours?
6. Let $\mathbf{V} = 2\mathbf{I} - 3\mathbf{J}$. Find the vector $\mathbf{W}$ to the left of $\mathbf{V}$, of the same length as $\mathbf{V}$, and making an angle of 60° with $\mathbf{V}$.
7. Find a base $\mathbf{L}$, $\mathbf{M}$ (in a base, the vectors are of length 1 and orthogonal) so that $\mathbf{L}$ is on the line $y = 5x$ and $\mathbf{M}$ is left of $\mathbf{L}$. Any vector $\mathbf{X} = x\mathbf{I} + y\mathbf{J}$ can be written in the form $\mathbf{X} = u\mathbf{L} + v\mathbf{M}$. Find u , v as functions of x, y .
8. What is the distance between the two parallel lines $L_1 : x + 2y = 7$, $L_2 : x + 2y = 11$?
9. Find the distance from the point $P(3, -2)$ to the line $L : 2x - y = 10$.
10. Find a point (x, y) on the line $2x - y = 10$ such that the triangle with vertices $(0, 0)$, $(5, 0)$, (x, y) has area equal to 15. How many such points are there?