Math 2210 - 9 Exam #2

roblem 1

$$f(x,y) = -2\sin(2x+y) - 6\cos(x-y)$$

$$f_x(x,y) = -4\cos(2x+y) + 6\sin(x-y) \quad (5\text{pts})$$

$$f_y(x,y) = -2\cos(2x+y) - 6\sin(x-y) \quad (5\text{pts})$$

$$f_{xx}(x,y) = 8\sin(2x+y) + 6\cos(x-y) \quad (5\text{pts})$$

$$f_{yy}(x,y) = 2\sin(2x+y) + 6\cos(x-y)$$

$$f_{x,y}(x,y) = 4\sin(2x+y) - 6\cos(x-y) \quad (5\text{pts})$$

$$f_{yx}(x,y) = 4\sin(2x+y) - 6\cos(x-y)$$

Problem 2.

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial s} \quad (5\text{pts})$$

$$= (2x+y)(t) + (2y+x)\cdot(1) + 2z\cdot(1) \quad (5\text{pts})$$

$$= 2xt + yt + 2y + x + 2z \quad (5\text{pts})$$

$$= 2st^2 + st - t^2 + 2s - 2t + st + 2s + 4t$$

$$= 2st^2 + 2st - t^2 + 4s + 2t. \quad (5\text{pts})$$

Problem 3

$F(x,y) = 3x^3 + y^2 - 9x + 4y$

$F_x(x,y) = 9x^2 - 9 = 0$ $\quad x = \pm 1$ $\quad$ ~~(5pts)~~

$F_y(x,y) = 2y + 4 = 0$ $\quad y = -2$ $\quad$ (5pts)

$(1,-2)$ $\quad (-1,-2)$ ← critical points

$F_{xx} = 18x$ $\quad F_{yy} = 2$ $\quad F_{xy} = 0$

(5pts)

$D(x,y) = F_{xx}F_{yy} - F_{xy}^2$

$F_{xx}(1,-2) = 18 > 0$

So $D(1,-2) = 18 \cdot 2 - 0 = 36 > 0$ $\quad$ Minimum. (5pt

$F(1,-2) = 3\cdot(1)^3 + (-2)^2 - 9\cdot(1) + 4(-2) = -10$.

$D(-1,-2) = -18 \cdot 2 - 0 = -36 < 0$ $\quad$ Saddle.

(5pts).

Problem 4

$$\frac{\partial T}{\partial x} = ye^{xy} - y^2 - 2xyz$$

$$\frac{\partial T}{\partial y} = xe^{xy} - 2xy - x^2 z$$

$$\frac{\partial T}{\partial z} = -x^2 y$$

(10pts)

$$\frac{\partial T}{\partial x}(0,-1,2) = -1-1 = -2$$

$$\frac{\partial T}{\partial y}(0,-1,2) = 0$$

$$\frac{\partial T}{\partial z}(0,-1,2) = 0$$

(5pts)

So $\nabla T = \langle -2, 0, 0 \rangle$

then the direction of the greatest <u>drop</u> is

$$-\nabla T = \langle 2, 0, 0 \rangle$$

(5pts)

Bonus :

$$D_{\vec{u}} T = \vec{u} \cdot \nabla T = \langle 1, 0, 0 \rangle \cdot \langle -2, 0, 0 \rangle = \boxed{-2}$$

directional derivative

(5pts)

tangent plane @ $P(0, -1, 2)$

$$\left\langle \frac{\partial T}{\partial x}(p), \frac{\partial T}{\partial y}(p), \frac{\partial T}{\partial z}(p) \right\rangle \cdot \langle x-0, y-(-1), z-2 \rangle = 0$$

$$\langle -2, 0, 0 \rangle \langle x, y+1, z-2 \rangle = 0$$

plane : $\boxed{-2x = 0}$ or $\boxed{x = 0}$ (5pts)

Problem 5

$f_x(x,y) = y-2x = 0$

$f_y(x,y) = x-2y = 0$

$\Rightarrow$ critical point $(0,0)$

$D(0,0) = (-2)^2 - 1^2 = 3 > 0$, $f_{xx}(0,0) = -2 < 0$

Local ~~possible~~ Maximum ~~extremal~~ value $f(0,0) = 4$ (5pts)

The boundary of S : $x = \cos\theta$ $y = \sin\theta$ $\theta \in [0, 2\pi]$

Let $g(\theta) = f(\cos\theta, \sin\theta)$, $\theta \in [0, 2\pi]$ (5pts)

$$g'(\theta) = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$= (y-2x)(-\sin\theta) + (x-2y)\cos\theta$$

$$= (\sin\theta - 2\cos\theta)(-\sin\theta) + (\cos\theta - 2\sin\theta)\cos\theta$$

$$= -\sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta \cancel{- 2\sin\theta\cos\theta}$$

$$= \cos 2\theta$$

critical points : $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (5pts)

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

Problem 5 (Continued)

$f\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = 3\frac{1}{2}$ $\qquad$ $f\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = 2\frac{1}{2}$

$f\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = 3\frac{1}{2}$ $\qquad$ $f\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = 2\frac{1}{2}$

So minimum value is $2\frac{1}{2}$ (5pts).