# MATH 2210-90 Fall 2011

## First Midterm Exam

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LAST NAME ______________________

FIRST NAME Grader's Copy

ID NO. ______________________

**INSTRUCTION:** SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

| | | |
|---|---|---|
| PROBLEM 1 | 20 | |
| PROBLEM 2 | 20 | |
| PROBLEM 3 | 20 | |
| PROBLEM 4 | 20 | |
| PROBLEM 5 | 20 | |
| TOTAL | 100 | |

# PROBLEM 1

(20 pt) Let $T$ be the triangle with vertices at $P = (-8, -9)$, $Q = (-7, 10)$, $R = (-7, 1)$, find the area of $T$.

**Hint:** Let $u$ be the vector from $R$ to $P$ and $v$, from $R$ to $P$ $\to Q$ Find $\|\vec{u} \times \vec{v}\|$. Do not calculate the numerical value.

typo

$\vec{RP} = \vec{u} = (-1, -10)$

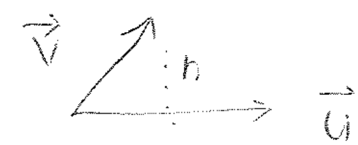
(5 pt)

$\vec{RQ} = \vec{v} = (0, 9)$

$\vec{u} \times \vec{v} = \begin{vmatrix} -1 & -10 \\ 0 & 9 \end{vmatrix}$ (5 pt)

$= (-1) \cdot 9 - 0(-10) = 1$ (5 pt)

$\cancel{\|\vec{u} \times \vec{v}\| = \frac{1}{2}}$

(  )



$h = \vec{v} \cdot \hat{u} \sin\theta$

Area of the triange $= \frac{1}{2}$

only fo oxplanati

(5 pt)

# PROBLEM 2

(20 pt) What is the distance from $(0, 3, 7)$ to the $xz$-plane?

**Hint:** What is the normal vector of $xz$-plane?

(1) The normal unit vector of the xz-plane

$$= \pm (0, 1, 0) = \hat{n} \quad (5pt)$$

Choose a pt on xz-plane

Say $(0, 0, 0)$ (5pt)

$$\vec{u} = (0, 3, 7) - (0, 0, 0) \quad (5pt)$$

$$= (0, 3, 7)$$

$$\|\vec{u} \cdot \hat{n}\| = 3 \quad (5pt)$$

$$\cancel{\|\vec{u} \times \hat{n}\| = \left\| \begin{matrix} i & j & k \\ 0 & 3 & 7 \\ 0 & 1 & 0 \end{matrix} \right\| = \|-7i\|}$$

$$\cancel{= 7}$$

(2) It is okay that they use the formula on p 571.

# PROBLEM 3

(20 pt) Consider the vector functions

$X(t) = 8\hat{\mathbf{i}} + \cos(7t)\hat{\mathbf{j}}, \quad Y(t) = \sin(7t)\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \quad Z(t) = X(t) \times Y(t).$

Find $dZ(t)/dt$.

**Notation:** $\hat{\mathbf{i}} = (1,0,0)$, $\hat{\mathbf{j}} = (0,1,0)$, $\hat{\mathbf{k}} = (0,0,1)$

$$\frac{d}{dt} Z(t) = X'(t) \times Y(t) + X(t) \times Y'(t)$$

$$= (0, -7\sin(7t), 0) \times (0, \sin(7t), 3)$$

(10pt)

$$+ (8, \cos(7t), 0) \times (0, 7\cos(7t), 0)$$

$$\begin{vmatrix} i & j & k \\ 0 & -7\sin 7t & 0 \\ 0 & \sin(7t) & 3 \end{vmatrix} = -21 \sin(7t)\, \hat{i}$$

(5pt)

$$\begin{vmatrix} i & j & k \\ 8 & \cos(7t) & 0 \\ 7\cos(7t) \to 0 & 0 & 0 \end{vmatrix} = -\cancel{8}\, 7 \cos(7t)\, \hat{k}$$

(5pt)

# PROBLEM 4

(20 pt) Given that the acceleration vector is $-9\cos(-3t)\hat{\mathbf{i}} + 9\sin(3t)\hat{\mathbf{j}} + 3t\hat{\mathbf{k}}$, the initial velocity is $(1, 0, 1)$, and the initial position vector is $(1, 1, 1)$.
Find the formula of the velocity vector and the position vector.

$$\int -9\cos(-3t)\,dt = 3\sin(-3t) + C_1$$

$$\int 9\sin(3t)\,dt = -3\cos(3t) + C_2$$

$$\int 3t\,dt = \frac{3}{2}t^2 + C_3$$

10 pt)

$$1 = C_1, \quad 0 = -3 + C_2, \quad 1 = C_3$$

$$C_2 = 3$$

(10 pt)

$$\int \left(3\sin(-3t) + 1,\ -3\cos(3t) + 3,\ \frac{3}{2}t^2 + 1\right) dt = \vec{S}(t) - (1, 1, 1)$$

$$S(t) = \left(\cos(-3t) + t + 1,\ -\sin(3t) + 3t + 1,\ \frac{1}{2}t^3 + t + 1\right.$$

# PROBLEM 5

(20 pt) Let $P$ be the point $(10, -4, 2)$ in cartesian coordinates. Find the cylindrical coordinates and the spherical coordinates.

**Remark:** Do not calculate the numerical values.

cylindrical

$(10, -4, 2) = (r, \theta, 2)$

10pt

where $\theta$ is in the four quadrant

$r = \sqrt{10^2 + (-4)^2}$

$\theta = 2\pi - \arctan\left(\frac{4}{10}\right)$

$\left(\frac{3\pi}{2} \leq \theta < 2\pi\right)$

sphere $= (\rho, \theta, \varphi)$

pt

$\rho = \sqrt{10^2 + (-4)^2 + 2}$

$\theta$ is the same as above

$\varphi = \arccos\left(\frac{2}{\rho}\right)$ Note. $0 \leq \varphi \leq \pi$