MATH 2210-90 Fall 2011

Final Exam

INSTRUCTOR: H.-PING HUANG

Hint: do NOT calculate any numerical value, unless specified otherwise.

LAST NAME ______________________

FIRST NAME ______________________

ID NO. ______________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	30	______
PROBLEM 2	30	______
PROBLEM 3	30	______
PROBLEM 4	30	______
PROBLEM 5	30	______
PROBLEM 6	30	______
PROBLEM 7	30	______
PROBLEM 8	10	______
TOTAL	220	______

1. Fundamental Theorem For Line Integrals
Let C be a piecewise smooth curve give parametrically by $\mathbf{r} = \mathbf{r}(t)$, $a \leq t \leq b$, which begins at $\mathbf{a} = \mathbf{r}(a)$ and ends at $\mathbf{b} = \mathbf{r}(b)$. If f is continuously differentiable on an open set containing C, then

$$\int_C \bigtriangledown f(\mathbf{r}) \cdot d\mathbf{r} = f(\mathbf{b}) - f(\mathbf{a})$$

2. Green's Theorem
Let C be a smooth, simple closed curve that forms the boundary of a region S in the xy- plane. If $M(x,y)$ and $N(x,y)$ are continuous and have continuous partial derivatives on S and its boundary C, then

$$\int\int_S (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}) = \oint_C Mdx + Ndy$$

3. Gauss's Theorem
Let $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ be a vector field such that $M, N.P$ have continuous first-order partial derivatives on a solid S with boundary ∂S. If $\mathbf{n}$ denotes the outer unit normal to ∂S, then

$$\int\int_{\partial S} \mathbf{F} \cdot \mathbf{n} dS = \int\int\int_S \text{div } \mathbf{F} dV$$

In outher words, the flux of $\mathbf{F}$ across the boundary of a closed region the three space is the triple integral of its divergence over that region.

4. Stokes's Theorem
Let S, ∂S, and $\mathbf{n}$ be as indicated in the textbook, and suppose that $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is a vector field, with $M, N.P$ having continuous first-order partial derivatives on S and its boundary ∂S. If $\mathbf{T}$ denotes the unit tangent vector to ∂S, then

$$\oint_{\partial S} \mathbf{F} \cdot \mathbf{T} ds = \int\int_S (\text{curl}\mathbf{F}) \cdot \mathbf{n} dS$$

PROBLEM 1

(30 pt) Let T be the triangle with vertices at $P = (-8, -9)$, $Q = (-7, 10)$, $R = (-7, 1)$, find the area of T.

PROBLEM 2

(30 pt) If $w = x^2 + y^2 + z^2 + xy$, where $x = st$, $y = s - t$, $z = s + 2t$, find $\partial w / \partial s$.

PROBLEM 3

(30 pt) Evaluate the integral

$$\int_0^4 \int_{x/2}^2 e^{y^2}\, dy dx.$$

PROBLEM 4

(30 pt)
A. Compute the gradient vector fields of the following function:

$$f(x,y) = x^5y^7, \bigtriangledown f(x,y) = \underline{\quad\quad}\mathbf{i} + \underline{\quad\quad}\mathbf{j}$$

B. If C_1 is the parabola: $x = t$, $y = t^2$, $0 \le t \le 5$, find $\int_{C_1} \bigtriangledown f \cdot d\mathbf{X}$.
C. If C_2 is the straight line segment: $x = 5t^2$, $y = 25t$, $0 \le t \le 1$, find $\int_{C_2} \bigtriangledown f \cdot d\mathbf{X}$.

PROBLEM 5

(30 pt)

$$\mathbf{F}(x,y) = \frac{-y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j}$$

Let C be the circle $\mathbf{X}(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$, $0 \leq t \leq 2\pi$. Compute $\int_C \mathbf{F} \cdot d\mathbf{X}$.

PROBLEM 6

(30 pt) Let $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + 2z\mathbf{k}$ and S be the sphere $x^2 + y^2 + z^2 = 9$. Denote by $\mathbf{n}$ the outward unit normal vector to S. Compute $\int_S \mathbf{F} \cdot \mathbf{n} ds$.

PROBLEM 7

(30 pt) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 81$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

PROBLEM 8

(10 pt) The unit disc in the plane has boundary S: the circle, $x^2+y^2=1$. Find out

$$\frac{1}{2}\oint_S (-y dx + x dy).$$