Mathematics 2210 Calculus III
Practice Final Examination

1. Find the symmetric equations of the line through the point (3,2,1) and perpendicular to the plane $7x - 3y + z = 14$.

2. Find the equation of the plane through the points (0,-1,1), (1,0,1) and (1,2,2).

3. A particle moves through the plane as a function of time: $\mathbf{X}(t) = t^2\mathbf{I} + 2t^3\mathbf{J}$. Find the unit tangent and normal vectors, and the tangential and normal components of the acceleration.

4. A particle moves through space as a function of time:

$$\mathbf{X}(t) = \cos t\mathbf{I} + t\sin t\mathbf{J} + t\mathbf{K} \ .$$

For this motion, find $\mathbf{T}$, $\mathbf{N}$, the the tangential and normal components of the acceleration, and the curvature at time $t = 3\pi/2$.

5. The particle of problem 3 moves in opposition to the force field $\mathbf{F}(x, y, z) = x\mathbf{I} - y\mathbf{J} - \mathbf{K}$. How much work is required to move the particle from (1,0,0) to $(1, 0, 2\pi)$?

6. Find the critical points of

$$f(x, y) = 3xy + \frac{1}{x} - \ln y$$

in the first quadrant. Classify as local maximum or minimum or saddle point.

7. The temperature distribution on the surface $x^2 + y^2 + z^2 = 1$ is given by $T(x, y, z) = xz + yz$. Find the hottest spot.

8. What is the equation of the tangent plane to the surface $z^2 - 3x^2 - 5y^2 = 1$ at the point (1,1,3)?

9. Consider the surface Σ

$$f(x, y) = \frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1 \ .$$

a) At what points on Σ is the tangent plane parallel to the plane $2x + y - z = 1$?

b) What constrained optimization problem is solved by part a)?

10. Find the volume of the tetrahedron in the first octant bounded by the plane

$$\frac{x}{5} + \frac{y}{3} + \frac{z}{2} = 1 \ .$$

11. a) Find the volume of the solid in the first quadrant which lies over the triangle with vertices (0,0), (1,0), (1,3) and under the plane $z = 2x + 3y + 1$.

b) Find the area of that segment of the plane.

12. Find the area of the region in the first quadrant bounded by the parabolas

$$y^2 - x = 1, \quad y^2 - x = 0, \quad y^2 + x = 5, \quad y^2 + x = 4 \ .$$

13. Find the mass of a lamina over the domain in the plane $D: \ 0 \leq y \leq x(1-x)$, if the density function is $\delta(x, y) = 1 + x + y$.

14. Find the center of mass of the piece of the unit sphere in the first octant:

$$x^2 + y^2 + z^2 \leq 1 \ , \qquad x \geq 0, \quad y \geq 0, \quad z \geq 0 \ .$$

15. Let

$$f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x} \ .$$

Find a) ∇f, b) curl ∇f, c) div ∇f, d) $\nabla(\text{div } \nabla f)$.

16. Let $\mathbf{F} = (y + 2xz)\mathbf{I} + (x + z^2 + 1)\mathbf{J} + (2yz + x^2\mathbf{K}$. Find a function f such that $\mathbf{F} = \nabla f$.

17. Let C be the curve in space given parametrically by the equations

$$x = t^2 - 3t + 5 \ , \quad y = (t^3 - 2)^2 \ , \quad z = t^4 + t^3 - t^2, 0 \leq t \leq 1 \ ,$$

and $\mathbf{F}$ the vector field

$$\mathbf{F}(x, y, z) = x\mathbf{I} + z\mathbf{J} + y\mathbf{K} \ .$$

What is $\int_C \mathbf{F} \cdot d\mathbf{X}$?

18. Let C be the curve given in polar coordinates by $r = 1 + \cos\theta, \ 0 \leq \theta \leq 2\pi$. Calculate $\int_C x dy$.

19. Let C be the part of the curve $y = x^2(24 - x)$ which lies in the first quadrant. Consider it directed from the point (0,0) to the point (24,0). Calculate

$$\int_C (y + 1)dx - xdy \ .$$