Calculus III
Practice Exam 3

1. Find the volume under the plane $z = x + 2y + 1$ over the triangle bounded by the lines $y = 0$, $x = 1$, $y = 2x$.

2. Let R be the region in the plane bounded by the curves $x = y^2$, $x = 3 - 2y^2$. Calculate

$$I = \int\int_R (y^2 - x)dx\ .$$

3. Let R be the region in the first quadrant bounded by the curves $y = x$ and $y = x^3$. What are the coordinates of its centroid?

4. What is the mass of the lamina bounded by the curves $y = 3x$ and $y = 6x - x^2$ where the density function is $\delta(x,y) = xy$?

5. As (u,v) runs through the region $u^2 + v^2 \leq 1$, the vector function

$$\mathbf{X}(u,v) = (u^2 + v^2)\mathbf{I} + (u^2 - v^2)\mathbf{J} + uv\mathbf{K}$$

describes a surface S in three space. Write down the double integral which must be calculated to find the surface area of S.

6. Find the volume of the region bounded below by the surface $z = 4x^2 + 25y^2$, and above by the plane $z = 100$.

7. Find the centroid of the region under the cone $z^2 = x^2 + y^2$ lying over the disc $x^2 + y^2 \leq 9$.

8. Find the volume inside the hyperboloid $x^2 + y^2 - z^2 = 1$, for $0 \leq z \leq 2$.

9. Find the surface area of the piece of the paraboloid $z = x^2 + y^2$ lying between the planes $z = 0$, $z = 2$.

10. The part R of the sphere of radius 1 centered at the origin which lies in the first octant is filled with a material whose density function is $\delta(x,y,z) = z^2 + xy$. Find the mass of this object.