

Calculus III
Practice Exam 2

1. A conic in the plane is given by the equation

$$x^2 - 2xy + y^2 + 2x - y = 0.$$

- a) What conic is it?
- b) At what angle to the x -axis are the axes of the conic?

2. A conic in the plane is given by the equation

$$5x^2 - xy + y^2 = 50.$$

- a) What conic is it?
- b) At what angle to the x -axis are the axes of the conic?

3. A parabola has its vertex at the origin, and its focus at the point $(3,4)$. Give the equation of the parabola. Recall that, for a parabola with vertex at the origin and focus at the point $(p,0)$, the equation is $y^2 = 4px$.

4. Let $f(x,y) = 3x^2y + 3xy$.

- a) $\nabla f = ?$
- b) What is the direction of maximum increase of f at the point $(1,2)$?
- c) What are the critical points of f ? What kind of critical points are they?

5. Let

$$f(x,y) = \frac{1}{x} + \frac{1}{y}.$$

- a) What is the tangent line to the curve $f(x,y) = 5/6$ at the point $(2,3)$?
- b) Find the equation of the tangent plane to the surface $z = f(x,y)$ at the point $(2,3,5/6)$.

6. Let

$$f(x,y,z) = \frac{1}{xy} + \frac{1}{yz}.$$

What is the equation of the tangent plane to the level surface $f(x,y,z) = 1$ at the point $(1,2,1)$?

7. Let $w = x\sqrt{y} + y\sqrt{z}$, and let γ be the curve $x = -t$, $y = t^2$, $z = 1 + t$, for $t > 0$. What is dw/dt at $t = 1$?

8. Let

$$f(x,y) = x^3y + \frac{1}{2}y^2x + yx^2.$$

Find all saddle points of the surface $z = f(x,y)$.

9. Find the point on the curve $2(x-1)^2 + 3y^2 = 22$ which is closest to the origin.

10. A rectangular box of maximum volume is to be constructed, with sides parallel to the coordinate planes, one corner at the origin and the diagonally opposite corner on the plane $2x + 3y + z = 1$. What are the dimensions of the box?