Calculus III
Practice Exam 1

1. $\mathbf{V} = 4\mathbf{I} + 7\mathbf{J},\ \mathbf{W} = -\mathbf{I} + 5\mathbf{J}$ are two vectors in the plane.
a) Find the angle between $\mathbf{V}$ and $\mathbf{W}$.
b) Find the angle between $\mathbf{V}$ and $\mathbf{W}^{\perp}$.
c) Find the area of the parallelogram spanned by $\mathbf{W}$ and $\mathbf{W}^{\perp}$.

2. Find the distance of the point (2,-2) from the line given by the equation $x + 2y = 8$.

3. A particle moves in the plane according to the equation

$$\mathbf{X}(t) = t\mathbf{I} - t^3\mathbf{J}$$

Find the velocity, speed, acceleration, tangent and normal vectors, and normal acceleration of the particle at any time t.

4. Find the symmetric equation of the line through the point (2,-1,3) which is perpendicular to the vectors $\mathbf{I} - 2\mathbf{J} + 3\mathbf{K}$ and $3\mathbf{I} - 2\mathbf{J} + \mathbf{K}$.

5. Find the equation of the plane through the origin which is normal to the line given parametrically by

$$\mathbf{X} = (3\mathbf{I} + 2\mathbf{J} - \mathbf{K}) + t(-\mathbf{I} + \mathbf{J} + 2\mathbf{K})\ .$$

6. Find a vector normal to the plane through $(0,0,0),\ (1,0,-1),\ (0,1,1)$.

7. Consider two different, but parallel planes given by the equations

$$\Pi_1\ :\quad (\mathbf{X} - \mathbf{X_1}) \cdot \mathbf{N} = 0\ , \qquad \Pi_2\ :\quad (\mathbf{X} - \mathbf{X_2}) \cdot \mathbf{N} = 0\ .$$

Express the distance between the planes as a function of $\mathbf{X_1},\ \mathbf{X_2}, \mathbf{N}$.

8. Find the distance of the point $(3,2,1)$ from the line whose symmetric equations are

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-1}{-2}$$

9. A particle moves in space according to the formula $\mathbf{X}(t) = \mathbf{I} + t\mathbf{J} - t^2\mathbf{K}$. Find the tangential and normal accelerations as functions of t. 10. A particle moves in space according to the formula $\mathbf{X}(t) = e^t\mathbf{I} + e^{2t}\mathbf{J} - t\mathbf{K}$.

Find the normal acceleration at the point $t = 0$.