Mathematics 2210 Calculus III, Final Examination Jul 29,30,2003

You may use graphing calculators and tables of Integrals. You **MUST** show enough work to convince me that you know how to do the problems.

1. Find the equation of the line through the origin and orthogonal to the plane through the points $P(2,-1,0), Q(1,2,1)$ and $R(3,0,2)$.

2. A particle moves through space as a function of time: $\mathbf{X}(t) = t\mathbf{I} + \ln t\mathbf{J} + t^2\mathbf{K}$. Find the tangential and normal components of the acceleration when $t = 1$.

3. Find the equation of the tangent plane to the surface $x^2 + y^2 + 3xy + 2xz = 5$ at the point (1,-1,3).

4. Find the maximum value of $3x + 2y + z$ on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$.

5. Find the center of mass of the lamina in the upper half plane bounded by the circle $x^2 + y^2 = 1$ if the density is $\delta(x, y) = x^2 + y^2$.

6. Given the vector field $\mathbf{F}(x, y) = (y-x^3)\mathbf{I}+(x-y^3)\mathbf{J}$, a) find a function f whose gradient is $\mathbf{F}$. b) Calculate div $\mathbf{F}$.

7. Let D be the region in the upper half plane bounded by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 9$. Let C be the boundary of D traversed counterclockwise. Find

$$\int_C y^2 dx + 3xy dy \ .$$

8. Let C be the circle $x^2 + y^2 = 16$, oriented counterclockwise. Calculate

$$\oint_C \frac{-y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy \ .$$