Mathematics 2210 Calculus III

Final Examination December 10,11,2003

You may use graphing calculators and tables of Integrals. You **MUST** show enough work to convince me that you know how to do the problems.

Part I. Do FIVE (5) of these first 7 problems.

1. Find the distance of the point (2,1,1) from the plane given by the equation $x+y-z=1$.

2. A particle moves in the plane as a function of time: $\mathbf{X}(t)=(2t+1)\mathbf{I}+(t^2-2t+3)\mathbf{J}$. Find the tangential and normal components of the acceleration.

3. Find the points where the ellipse $x^2+2xy+10y^2=63$ has a horizontal tangent.

4. Let $f(x,y)=x^2+xy+y^2$. A particle is moving through the plane so that its position at time t is $\mathbf{X}(t)=\sin t\mathbf{I}+\cos t\mathbf{J}$. Find df/dt when $t=\pi/3$.

5. Find a vector perpendicular to the line given by the symmetric equations

$$\frac{x-1}{2}=\frac{y+1}{4}=\frac{z}{5}$$

lying in the plane given by the equation $2x+y+z=0$.

6. Find the maximum value of $3x^2-y^2+z$ on the ellipsoid $x^2+2y^2+3z^2=1$.

7. Let R be the part of the unit sphere in the first octant. Suppose that it is filled with a material whose density at the point (x,y,z) is given by $\delta(x,y,z)=xyz$. Find the total mass.

Part II. Do ALL three problems.

8. Given the vector field $\mathbf{F}(x,y)=(x^2-y)\mathbf{I}+(y^2-x)\mathbf{J}$,
a) find a function f whose gradient is $\mathbf{F}$.

b) Calculate div $\mathbf{F}$.

9. Let D be the region in the plane bounded by the circle $x^2+y^2=9$. Let C be the boundary of D traversed counterclockwise. Find

$$\int_C y^2dx+2xydy\ .$$

10. Let C be the curve given parametrically by

$$\mathbf{X}(t)=t\mathbf{I}+t^2\mathbf{J}+t^3\mathbf{K}$$

for t running from 0 to 2. For the vector field $\mathbf{F}=x\mathbf{I}+y\mathbf{J}+2x\mathbf{K}$, find $\int_C \mathbf{F}\cdot\mathbf{T}ds$.