

Calculus II
Exam 2, Summer 2003

You may use graphing calculators and a Table of Integrals. Each problem is worth 20 points. You MUST show your work. Just the correct answer is not sufficient for any points.

1. A conic in the plane is given by the equation

$$2x^2 - 2xy + y^2 + 2x - y = 100 .$$

- a) What conic is it?
b) At what angle to the x -axis are the axes of the conic?

2. Let $f(x,y) = x^2 - 3xy - y^2$.

- a) What is ∇f ?
b) At the point $(1,2)$, in which direction is f increasing most rapidly?
c) Find the equation of the plane tangent to the surface $z = f(x,y)$ at the point $(1,2,-9)$.

3. Let $f(x,y,z) = x \sin(yz)$. Let the curve γ be given by the equation $\mathbf{X}(t) = 2t\mathbf{I} - t\mathbf{J} + \mathbf{K}$. Let $g(t) = f(\mathbf{X}(t))$. Do the following calculations at the value $t = \pi/2$:

a) $\nabla f =$

b) $\frac{d\mathbf{X}}{dt} =$

c) $\frac{dg}{dt} =$

4. Let $f(x,y) = 3x^2y + 3xy$.

- a) $\nabla f =$
b) What are the critical points of f ?
c) What kind of critical points are they?

5. Find the maximum value of xy on the curve $x^2 + 2y^2 = 1$.