1. \( \mathbf{V} = 3\mathbf{I} - \mathbf{J}, \mathbf{W} = 2\mathbf{I} + 5\mathbf{J} \) are two vectors in the plane.

a) Find the angle between \( \mathbf{V} \) and \( \mathbf{W} \).

**Answer.** Let \( \alpha \) be the angle. Then

\[
\cos \alpha = \frac{\mathbf{V} \cdot \mathbf{W}}{||\mathbf{V}|| ||\mathbf{W}||} = \frac{3(2) - 5}{\sqrt{10} \sqrt{29}} = \frac{1}{\sqrt{290}} = 0.0587,
\]

so \( \alpha = 1.512 \) radians, or 86.63°.

b) Find the vector which is orthogonal to \( \mathbf{V} \) and counterclockwise from \( \mathbf{W} \).

**Answer.** Since \( \mathbf{W} \) is counterclockwise from \( \mathbf{V} \), and the angle \( \alpha \) is less than a right angle, the vector we seek is \( \mathbf{V} \times \mathbf{W} = 3\mathbf{J} \).

c) Find the area of the parallelogram spanned by \( \mathbf{V} \) and \( \mathbf{W} \).

**Answer.** The area is \( \mathbf{V} \times \mathbf{W} = 2 + 15 = 17. \)

2. A particle moves in the plane according to the equation

\[
\mathbf{X}(t) = \ln t \mathbf{I} + \frac{1}{t} \mathbf{J}
\]

Find the velocity, speed, acceleration, tangent and normal vectors, and normal acceleration of the particle at any time \( t \).

**Answer.** Differentiate \( \mathbf{X}(t) \):

\[
\mathbf{V} = \frac{1}{t} \mathbf{I} - \frac{1}{t^2} \mathbf{J} = \frac{1}{t^2} (t \mathbf{I} - \mathbf{J}),
\]

\[
\mathbf{A} = -\frac{1}{t^2} \mathbf{I} + \frac{2}{t^3} \mathbf{J},
\]

\[
\frac{ds}{dt} = \frac{1}{t^2 \sqrt{1 + t^2}}, \quad \mathbf{T} = \frac{t \mathbf{I} - \mathbf{J}}{\sqrt{1 + t^2}}, \quad \mathbf{N} = \frac{1 + t \mathbf{J}}{\sqrt{1 + t^2}},
\]

\[
a_N = \mathbf{A} \cdot \mathbf{N} = \frac{1}{\sqrt{1 + t^2}} \left(-\frac{1}{t^2} + \frac{2}{t^2} \right) = \frac{1}{t^2 \sqrt{1 + t^2}}.
\]

3. Find the equation of the plane through the point \((0,-1,3)\) which is parallel to the vectors \( \mathbf{I} - 2\mathbf{J} + 2\mathbf{K} \) and \( 3\mathbf{I} - 2\mathbf{J} + \mathbf{K} \).

**Answer.** The normal to the plane is the cross product of the two given vectors. This is

\[
\mathbf{N} = \det \begin{pmatrix} 1 & \mathbf{J} & \mathbf{K} \\ 1 & -2 & 2 \\ 3 & -2 & 1 \end{pmatrix} = 2\mathbf{I} + 5\mathbf{J} + 4\mathbf{K}.
\]

Taking \( \mathbf{X}_0 = (0)\mathbf{I} - \mathbf{J} + 3\mathbf{K} \), the equation is \( (\mathbf{X} - \mathbf{X}_0) \cdot \mathbf{N} = 0 \), which comes to \( 2x + 5y + 4z = 7 \).
4. Find the distance of the point \((2,0,1)\) from the line whose symmetric equations are

\[
\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-1}{-2}.
\]

**Answer.** Let \(Q\) be the given point. \(P = 2I - J + K\) is a point on the line, and \(L = 3I + 4J - 2K\) is a vector in the direction of the line. Then \(Q - P = -J\), and the desired distance is

\[
\frac{|-J \times L|}{|L|} = \sqrt{\frac{13}{29}}.
\]

5. A particle moves in space according to the formula

\[
X(t) = e^t I + e^{2t} J - tK.
\]

Find the normal acceleration at the point \(t = 0\).

**Answer.** Differentiate;

\[
V(t) = e^t I + 2e^{2t} J - K, \quad A(t) = e^t I + 4e^{2t} J.
\]

Now, evaluate at \(t = 0\) (don’t work with the general formulas!) to find

\[
V = I + 2J - K, \quad A = I + 4J.
\]

Then \(|V| = \sqrt{6}\) and since \(T = V / |V|\) we have

\[
A \cdot T = \frac{9}{\sqrt{6}},
\]

\[
A - (A \cdot T)T = -\frac{1}{2}I + J + \frac{3}{2}K,
\]

so \(a_N = |A - (A \cdot T)T| = \sqrt{7/2}\).