

Calculus III 2210-90 Exam 2
Summer 2014

Name _____

Instructions. Show all work and include appropriate explanations when necessary. Correct answers unaccompanied by work may not receive full credit. Please try to do all your work in the space provided and circle your final answers.

1. (14pts) For this problem, consider the function

$$f(x, y) = x^2 + \sin(xy).$$

- (a) (4pts) Find the gradient of f at $(1, 0)$, $\nabla f(1, 0)$.
- (b) (3pts) Find the maximum rate of change of f at the point $(1, 0)$.
- (c) (3pts) Let $\mathbf{u} = \langle -\frac{8}{17}, \frac{15}{17} \rangle$. Find $D_{\mathbf{u}}f(1, 0)$, that is, find the directional derivative of f at $(1, 0)$ in the direction of \mathbf{u} .
- (d) (4pts) Find the equation of the tangent plane to the surface $z = x^2 + \sin(xy)$ at the point $(1, 0, 1)$.
2. (10pts) Ohm's Law states that the current I through a simple circuit is equal to the voltage V over the resistance R , or $I = \frac{V}{R}$. Therefore, if 5 volts are applied to a circuit that has 1 ohm of resistance, 5 amps of current will flow. Use differentials to estimate how the current changes if the voltage increases from 5 to 5.1 volts and the resistance decreases from 1 to .8 ohms.

3. (8pts) Consider the function

$$f(x, y) = x^2e^{-y} + ye^{-y} + 9.$$

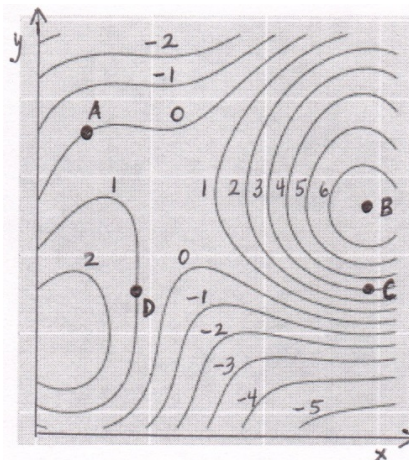
(a) (4pts) Find the critical point(s) of f .

(b) (4pts) Find the discriminant, $D = f_{xx}f_{yy} - (f_{xy})^2$, and use it to determine whether each of the critical points found in part (a) is a local minimum, a local maximum, or a saddle point.

4. (10pts) Use Lagrange multipliers to find the maximum and minimum **values** of the function $f(x, y) = x + 8y$ on the ellipse $\frac{x^2}{4} + y^2 = 1$.

5. (8pts) The picture below is a contour plot of the function $f(x, y)$, along with four points labeled **A-D**. That is, the curves are level curves of the function $f(x, y)$ corresponding to the values written next to the curves. Answer the following questions by writing one letter **A-D** in each blank.

- (a) _____ At what point is the value of f equal to 1?
- (b) _____ At what point is ∇f parallel to \mathbf{j} ?
- (c) _____ At what point is $\|\nabla f\|$ the largest?
- (d) _____ At what point is ∇f equal to zero?



6. (24pts) Evaluate the following integrals:

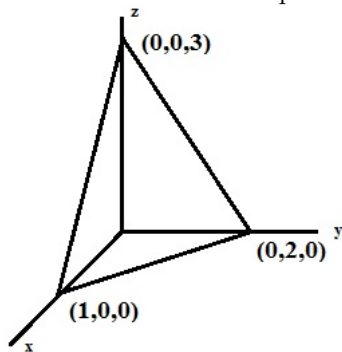
(a) (6pts) $\iint_R (x^2y + 3) dA$, where R is the rectangle $R = \{(x, y) | -1 \leq x \leq 1, 0 \leq y \leq 1\}$.

(b) (6pts) $\iint_T xy dA$, where T is the region in the xy -plane bounded by the x -axis, the curve $y = x^3$, and the line $x = 2$.

(c) (6pts) $\iint_E \sqrt[4]{x^2 + y^2} dA$, where E the annulus $4 \leq x^2 + y^2 \leq 9$.

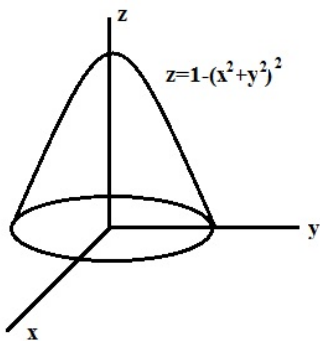
(d) (6pts) $\iiint_B (xy + z \sin(y)) dV$, where B is the box $B = \{(x, y, z) | 0 \leq x \leq 1, 0 \leq y \leq \pi, 0 \leq z \leq 2\}$.

7. (8pts) Use a triple integral to find the volume of the tetrahedron in the first octant determined by the coordinate planes and the plane $6x + 3y + 2z = 6$.



8. (8pts) Let G denote the region bounded by the surface $z = 1 - (x^2 + y^2)^2$ and the xy -plane. Compute

$$\iiint_G z \, dV.$$



9. (10pts) In this problem, we will compute the improper integral $\iiint_{\mathbb{R}^3} e^{-(x^2+y^2+z^2)^{3/2}} \, dV$, where \mathbb{R}^3 denotes all of 3-space.

- (a) (6pts) First compute $\iiint_{S_R} e^{-(x^2+y^2+z^2)^{3/2}} \, dV$, where S_R is the solid sphere of radius R , $S_R = \{(x, y, z) | x^2 + y^2 + z^2 \leq R^2\}$.

- (b) (4pts) Now take the limit of your answer in part (a) as $R \rightarrow \infty$.