

2210-90 Exam 2
Spring 2014

Name _____

Instructions. Show all work and include appropriate explanations when necessary. Correct answers unaccompanied by work may not receive full credit. Please try to do all your work in the space provided and circle your final answers.

1. (11pts) For this problem, consider the function

$$f(x, y) = x^2y^3 - ye^x + 9.$$

- (a) (4pts) Find the gradient of f at $(0, 2)$, $\nabla f(0, 2)$.

- (b) (3pts) Find the maximum rate of change of f at the point $(0, 2)$.

- (c) (3pts) Let $\mathbf{u} = \langle \frac{5}{13}, -\frac{12}{13} \rangle$. Find $D_{\mathbf{u}}f(0, 2)$, that is, find the directional derivative of f at $(0, 2)$ in the direction of \mathbf{u} .

2. (10pts) Consider the ellipsoid

$$3x^2 + y^2 + 2z^2 = 25.$$

- (a) Find a unit vector which is perpendicular to the tangent plane to the ellipsoid at the point $(1, 2, 3)$.

- (b) Find an equation for the tangent plane to the ellipsoid at $(1, 2, 3)$.

3. (8pts) Consider the function

$$f(x, y) = (x^2 + y^2)e^y.$$

- (a) (4pts) Find the two critical points of f .

- (b) (4pts) Find the discriminant, $D = f_{xx}f_{yy} - (f_{xy})^2$, and use it to determine whether each of the critical points found in part (a) is a local minimum, a local maximum, or a saddle point.

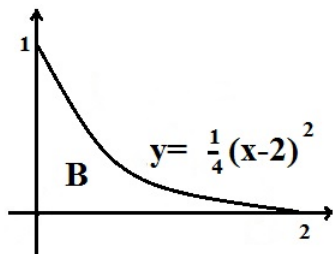
4. (10pts) Use Lagrange multipliers to find the maximum and minimum **values** of the function $f(x, y) = x^2y^2$ on the circle $x^2 + y^2 = 1$.

5. (6pts) The region B is bounded by the x -axis, the y -axis, and the curve $y = \frac{1}{4}(x-2)^2$. See the picture of B below. If I want to compute the integral of a function $f(x, y)$ over the region B , I can compute it in two equivalent ways:

$$\int_0^2 \int_0^{\frac{1}{4}(x-2)^2} f(x, y) \, dy \, dx = \int_a^b \int_{g_1(y)}^{g_2(y)} f(x, y) \, dx \, dy$$

where

$$a = \underline{\hspace{2cm}} \quad b = \underline{\hspace{2cm}} \quad g_1(y) = \underline{\hspace{2cm}} \quad g_2(y) = \underline{\hspace{2cm}}$$



6. (28pts) Find the following double integrals:

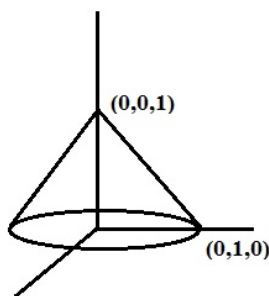
(a) (6pts) $\int_0^{\pi/2} \int_0^2 (x + \sin(2y)) \, dx \, dy$

(b) (6pts) $\iint_R \frac{y}{x^2} \, dA$, where R is the rectangle $R = \{(x, y) | 1 \leq x \leq 2, 0 \leq y \leq 1\}$.

(c) (8pts) $\iint_T x^2 y \, dA$, where T is the region in the xy -plane bounded by the x -axis and the curve $y = 4 - x^2$.

(d) (8pts) $\iint_D (x^2 + y^2)^{-1/2} \, dA$, where D the part of the disk $x^2 + y^2 \leq 16$ in the first quadrant.

7. (8pts) Use an integral in cylindrical coordinates to find the volume of the solid cone with radius of the base and height equal to 1. Recall that the volume of a region R can be computed as $\iiint_R 1 \, dV$.



8. (8pts) Let S denote the spherical shell $S = \{(x, y, z) | 1 \leq x^2 + y^2 + z^2 \leq 4\}$. Compute

$$\iiint_S \frac{1}{x^2 + y^2 + z^2} \, dV$$

9. (11pts) Consider the map from the uv -plane to the xy -plane given by

$$x(u, v) = u \quad y(u, v) = v + u^2$$

This map takes the unit square S in the uv -plane to the ‘curvy’ parallelogram R in the xy -plane. See picture below.

- (a) (4pts) Compute the Jacobian of this map. Recall that the Jacobian is the determinant $J(u, v) =$

$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}.$$

- (b) (7pts) Compute $\iint_R (2x + y) \, dA$ by using the change of variables formula

$$\iint_R f(x, y) \, dx \, dy = \iint_S f(x(u, v), y(u, v)) |J(u, v)| \, du \, dv$$

