

2210-90 Exam 2  
Spring 2013

Name \_\_\_\_\_

**Instructions.** Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (12pts, 4pts each) Evaluate the following limits. If the limit does not exist, write 'DNE' as your answer.

(a)  $\lim_{(x,y) \rightarrow (1,0)} \frac{\cos(xy)}{1+x^2-y^2}$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x-2y}{x+y}$

(c)  $\lim_{(x,y) \rightarrow (0,0)} \frac{y^4}{x^2+y^2}$

2. (12pts, 4pts each) Find the indicated partial derivatives of the function  $f(x, y) = \sin(xy + y^2)$ .

(a)  $f_x(x, y) =$

(b)  $f_y(x, y) =$

(c)  $f_{xy}(x, y) =$

3. (13pts) For this problem, consider the function

$$f(x, y) = xy^2 + x^3 - xy^3.$$

(a) (4pts) Find  $\nabla f(1, 1)$ , that is, find the gradient of  $f$  at  $(1, 1)$ .

(b) (3pts) Find the maximum rate of change of  $f$  at  $(1, 1)$ .

(c) (3pts) Let  $\mathbf{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$ . Find  $D_{\mathbf{u}}f(1, 1)$ , that is, find the directional derivative of  $f$  at  $(1, 1)$  in the direction of  $\mathbf{u}$ .

(d) (3pts) Find the unit vector that indicates the direction in which  $f$  increases the fastest at  $(1, 1)$ .

4. (12pts) Find the equation for that tangent plane to the ellipse

$$3x^2 + y^2 + 2z^2 = 6$$

at the point  $(1, -1, -1)$ .

5. (11pts) Consider the function

$$f(x, y) = x^3 + y^2 - 3x + 6y.$$

(a) (4pts) Find the 2 critical points of  $f$ .

(b) (3pts) Find the discriminant,  $D = f_{xx}f_{yy} - (f_{xy})^2$ .

(c) (4pts) Use the discriminant to determine whether each of the critical points found in part (a) is a local minimum, a local maximum, or a saddle point.

6. (12pts) Use the method of Lagrange multipliers to find the maximum and minimum values of the function  $f(x, y) = x^2 + y^2 - 2x - 2y$  on the circle  $x^2 + y^2 = 2$ .

7. (28pts) Find the following double integrals:

(a) (6pts)  $\int_1^3 \int_0^2 (xy - y^2) dx dy$

(b) (6pts)  $\iint_R (x + 4y) dA$ , where  $R$  is the rectangle  $R = \{(x, y) | 0 \leq x \leq 2, 0 \leq y \leq 1\}$ .

(c) (8pts)  $\iint_T xy dA$ , where  $T$  is the triangle with vertices  $(0, 0)$ ,  $(3, 0)$ , and  $(3, 6)$ .

(d) (8pts)  $\iint_D \frac{xy}{x^2 + y^2} dA$ , where  $D$  is the piece of the unit disk in the first quadrant, i.e. the set of points  $(x, y)$  where  $x^2 + y^2 \leq 1$  and both  $x$  and  $y$  are greater than or equal to zero.

