2210-90 Exam 1
Summer 2014

Name _____

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (14pts) Consider the vectors $\mathbf{u} = \langle -1, 5, 2\rangle$ and $\mathbf{v} = \langle 2, 2, -3\rangle$. Find

   (a) (2pts) $2\mathbf{u} - 3\mathbf{v}$

   (b) (2pts) $||\mathbf{u}||$

   (c) (2pts) $\mathbf{u} \cdot \mathbf{v}$

   (d) (4pts) the scalar projection of $\mathbf{u}$ onto $\mathbf{v}$. Recall, this is the dot product of $\mathbf{u}$ with the unit vector pointing in the same direction as $\mathbf{v}$.

   (e) (4pts) $\mathbf{u} \times \mathbf{v}$

2. (8pts) Find an equation of the plane containing the point $(0, 5, -4)$ which is perpendicular to the line

$$\mathbf{r}(t) = \langle 1, -1, 4\rangle + t\langle 3, -2, -1\rangle$$

3. (6pts) Find the equation of the largest sphere centered at $(2, 3, 5)$ that is completely contained in the first octant. **Note:** the first octant is where $x \geq 0$, $y \geq 0$, and $z \geq 0$.

4. (17pts) Suppose a particle's position at time t is given by the curve

$$\mathbf{r}(t) = \sin t\mathbf{i} - 5t\mathbf{j} - \cos t\mathbf{k}.$$

(a) (2pts) Find the velocity $\mathbf{v}(t) = \mathbf{r}'(t)$ of the particle at time t.

(b) (3pts) Find the arc length of the curve between times $t = 0$ and $t = 3$.

(c) (2pts) Find the acceleration $\mathbf{a}(t) = \mathbf{r}''(t)$ of the particle at time t.

(d) (2pts) Find the unit tangent vector $\mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||}$.

(e) (3pts) Find the principal unit normal vector $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}$.

(f) (5pts) Find the curvature $\kappa(t) = \frac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$ of the particle's path at time t.

5. (12 pts) Match the equation with the type of surface it describes by writing the appropriate capital letter (**A-F**) in the provided blank. Each answer will be used exactly once.

(a) _____ $x^2 + y^2 - z^2 = 1$

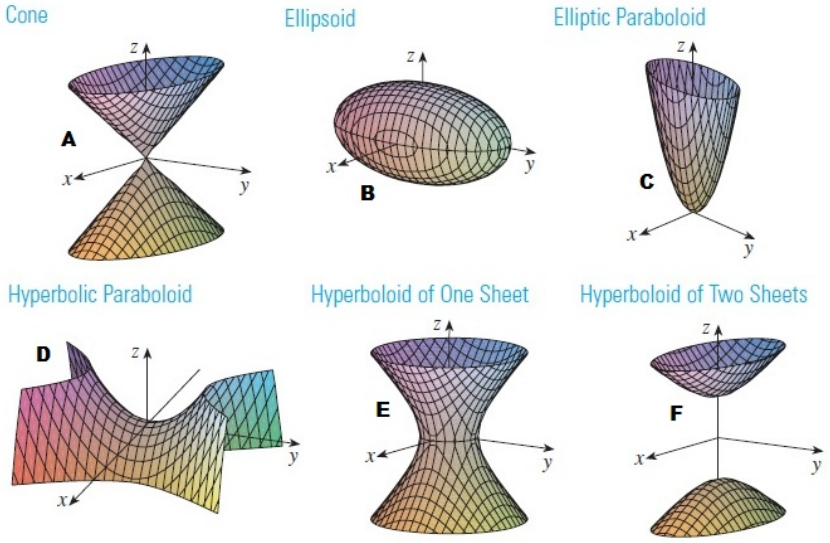
(b) _____ $3x^2 + y^2 + 3z^2 = 1$

(c) _____ $x^2 + y^2 - z^2 = -1$

(d) _____ $3x^2 + y^2 - z = 0$

(e) _____ $x^2 + 2y^2 - z^2 = 0$

(f) _____ $-x^2 + y^2 - z = 0$



(Images taken from page 808 of Stewart, *Calculus-Early Transcendentals*, 6e)

6. (10pts) Match the function with the description of its level sets (z = constant) by writing the appropriate capital letter (**A-E**) in the provided blank. Each letter should be used exactly once.

_____	$z = x^2 + y^2$	**A** a collection of parallel lines
_____	$z = \sqrt{x^2 + 2y^2 - 1}$	**B** a collection of concentric circles
_____	$z = \frac{y}{x}$	**C** a collection of hyperbolas
_____	$z = x^2 - y^2$	**D** a collection of lines through the origin
_____	$z = 3x - 2y$	**E** a collection of ellipses

7. (9pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated

(a) Find the Cartesian coordinates of the point with spherical coordinates $(\rho, \theta, \phi) = (\sqrt{2}, \frac{3\pi}{4}, \frac{\pi}{2})$

$x =$ _____ $y =$ _____ $z =$ _____

(b) Find the Cartesian coordinates of the point with cylindrical coordinates $(r, \theta, z) = (5, \frac{\pi}{6}, -2)$

$x =$ _____ $y =$ _____ $z =$ _____

(c) Find the cylindrical coordinates of the point with Cartesian coordinates $(x, y, z) = (3, -3, 1)$

$r =$ _____ $\theta =$ _____ $z =$ _____

8. (12pts) Evaluate the following limits. Show your work. If they do not exist, write 'DNE' and explain why.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{e^{x+y}+6}{1-x}$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{2x-3y}{x-y}$

(c) $\lim_{(x,y)\to(0,0)} \dfrac{x^3-2y^4}{x^2+y^2}$ **Hint:** Use polar coordinates.

9. (12pts) Consider the function

$$f(x,y) = y^2 \sin x + y^3 - x\cos y.$$

(a) (6pts) Find the equation of the tangent plane to the graph of $z = f(x,y)$ at the point $(0,\pi,\pi^3)$.

(b) (6pts) Find the following second derivatives:

i. $f_{xx}(x,y) =$ _____

ii. $f_{yy}(x,y) =$ _____

iii. $f_{xy}(x,y) =$ _____