Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (14pts) Consider the vectors $\mathbf{u} = \langle -1, 5, 2 \rangle$ and $\mathbf{v} = \langle 2, 2, -3 \rangle$. Find

(a) (2pts) $2\mathbf{u} - 3\mathbf{v}$

(b) (2pts) $||\mathbf{u}||$

(c) (2pts) $\mathbf{u} \cdot \mathbf{v}$

(d) (4pts) the scalar projection of $\mathbf{u}$ onto $\mathbf{v}$. Recall, this is the dot product of $\mathbf{u}$ with the unit vector pointing in the same direction as $\mathbf{v}$.

(e) (4pts) $\mathbf{u} \times \mathbf{v}$

2. (8pts) Find an equation of the plane containing the point $(0, 5, -4)$ which is perpendicular to the line

$$\mathbf{r}(t) = \langle 1, -1, 4 \rangle + t\langle 3, -2, -1 \rangle$$
3. (6pts) Find the equation of the largest sphere centered at (2, 3, 5) that is completely contained in the first octant. **Note:** the first octant is where \( x \geq 0, y \geq 0, \) and \( z \geq 0. \)

4. (17pts) Suppose a particle’s position at time \( t \) is given by the curve
\[
r(t) = \sin t \mathbf{i} - 5t \mathbf{j} - \cos t \mathbf{k}.
\]
(a) (2pts) Find the velocity \( \mathbf{v}(t) = r'(t) \) of the particle at time \( t. \)

(b) (3pts) Find the arc length of the curve between times \( t = 0 \) and \( t = 3. \)

(c) (2pts) Find the acceleration \( \mathbf{a}(t) = r''(t) \) of the particle at time \( t. \)

(d) (2pts) Find the unit tangent vector \( \mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||}. \)

(e) (3pts) Find the principal unit normal vector \( \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}. \)

(f) (5pts) Find the curvature \( \kappa(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3} \) of the particle’s path at time \( t. \)
5. (12 pts) Match the equation with the type of surface it describes by writing the appropriate capital letter (A-F) in the provided blank. Each answer will be used exactly once.

(a) _____  $x^2 + y^2 - z^2 = 1$
(b) _____  $3x^2 + y^2 + 3z^2 = 1$
(c) _____  $x^2 + y^2 - z^2 = -1$
(d) _____  $3x^2 + y^2 - z = 0$
(e) _____  $x^2 + 2y^2 - z^2 = 0$
(f) _____  $-x^2 + y^2 - z = 0$

6. (10pts) Match the function with the description of its level sets ($z = \text{constant}$) by writing the appropriate capital letter (A-E) in the provided blank. Each letter should be used exactly once.

_____  $z = x^2 + y^2$  \hspace{1cm} \textbf{A} \hspace{1cm} \text{a collection of parallel lines}
_____  $z = \sqrt{x^2 + 2y^2 - 1}$  \hspace{1cm} \textbf{B} \hspace{1cm} \text{a collection of concentric circles}
_____  $z = \frac{y}{x}$  \hspace{1cm} \textbf{C} \hspace{1cm} \text{a collection of hyperbolas}
_____  $z = x^2 - y^2$  \hspace{1cm} \textbf{D} \hspace{1cm} \text{a collection of lines through the origin}
_____  $z = 3x - 2y$  \hspace{1cm} \textbf{E} \hspace{1cm} \text{a collection of ellipses}

7. (9pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated

(a) Find the Cartesian coordinates of the point with spherical coordinates $(\rho, \theta, \phi) = (\sqrt{2}, \frac{3\pi}{4}, \frac{\pi}{2})$

\hspace{1cm} $x = \underline{\quad} \hspace{1cm} y = \underline{\quad} \hspace{1cm} z = \underline{\quad}$

(b) Find the Cartesian coordinates of the point with cylindrical coordinates $(r, \theta, z) = (5, \frac{\pi}{6}, -2)$

\hspace{1cm} $x = \underline{\quad} \hspace{1cm} y = \underline{\quad} \hspace{1cm} z = \underline{\quad}$

(c) Find the cylindrical coordinates of the point with Cartesian coordinates $(x, y, z) = (3, -3, 1)$

\hspace{1cm} $r = \underline{\quad} \hspace{1cm} \theta = \underline{\quad} \hspace{1cm} z = \underline{\quad}$
8. (12pts) Evaluate the following limits. Show your work. If they do not exist, write ‘DNE’ and explain why.

(a) \[ \lim_{{(x,y) \to (0,0)}} \frac{e^{x+y} + 6}{1-x} \]

(b) \[ \lim_{{(x,y) \to (0,0)}} \frac{2x - 3y}{x - y} \]

(c) \[ \lim_{{(x,y) \to (0,0)}} \frac{x^3 - 2y^4}{x^2 + y^2} \]

**Hint:** Use polar coordinates.

9. (12pts) Consider the function \[ f(x,y) = y^2 \sin x + y^3 - x \cos y. \]

(a) (6pts) Find the equation of the tangent plane to the graph of \( z = f(x,y) \) at the point \((0, \pi, \pi^3)\).

(b) (6pts) Find the following second derivatives:

i. \( f_{xx}(x,y) = \) __________________________

ii. \( f_{yy}(x,y) = \) __________________________

iii. \( f_{xy}(x,y) = \) __________________________