Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (16pts) Consider the vectors \( \mathbf{u} = \langle 6, 0, 2 \rangle \) and \( \mathbf{v} = \langle -1, 7, 3 \rangle \). Find
   (a) (2pts) \( \mathbf{v} - 2\mathbf{u} \)
   (b) (2pts) \( ||\mathbf{u}|| \)
   (c) (2pts) The unit vector which points in the same direction as \( \mathbf{u} \)
   (d) (2pts) \( \mathbf{u} \cdot \mathbf{v} \)
   (e) (1pt) Are \( \mathbf{u} \) and \( \mathbf{v} \) orthogonal? Circle one: \( YES \quad NO \)
   (f) (3pts) \( \mathbf{u} \times \mathbf{v} \)
   (g) (4pts) Two of the following quantities are zero (or the zero vector). Which ones? Circle two letters.
      A. \( \mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) \)          B. \( \mathbf{u} \times \mathbf{u} \)          C. \( \mathbf{u} \cdot \mathbf{u} \)          D. \( \mathbf{v} \times (\mathbf{u} \times \mathbf{v}) \)

2. (7pts) Find an equation of the plane consisting of all points that are equidistant from the points \( P = (1, 0, -1) \) and \( Q = (3, 2, 1) \).
3. (17pts) Suppose a particle’s position at time $t$ is given by the curve

$$
r(t) = (\cos t + t \sin t)i + 4t^2j + (\sin t - t \cos t)k.
$$

For this problem, it is helpful if you remember the trig identity $\sin^2 t + \cos^2 t = 1$.

(a) (2pts) Find the velocity $v(t)$ of the particle at time $t$.

(b) (3pts) Find the arc length of the curve between times $t = 0$ and $t = 2$.

(c) (2pts) Find the acceleration $a(t)$ of the particle at time $t$.

(d) (2pts) Find the unit tangent vector $T(t) = \frac{v(t)}{||v(t)||}$.

(e) (3pts) Find the principal unit normal vector $N(t) = \frac{T'(t)}{||T'(t)||}$.

(f) (5pts) Find the curvature $\kappa(t)$ of the particle’s path at time $t$.

4. (5pts) Suppose the acceleration of a particle is given by

$$
a(t) = (2t, t + \sin t, e^{-t}).
$$

If the particle’s initial velocity is $v(0) = (2, -3, 1)$, what is the velocity of the particle at time $t$?
5. (14pts) Match the equation with the type of surface it determines by writing the appropriate capital letter (A-G) in the provided blank. Each letter should be used exactly once.

\[
\begin{align*}
A & : x^2 + y^2 + z^2 = 1 & \text{Elliptic Paraboloid} \\
B & : x^2 + z^2 - y^2 = 1 & \text{Ellipsoid} \\
C & : x^2 + 2y^2 + 3z^2 = 1 & \text{Hyperboloid of one sheet} \\
D & : x^2 - 2y^2 - z = 0 & \text{Hyperboloid of two sheets} \\
E & : y = x + 3z - 7 & \text{Hyperbolic Paraboloid} \\
F & : x^2 + 2y^2 - z = 0 & \text{Plane} \\
G & : z^2 - x^2 - y^2 = 1 & \text{Sphere}
\end{align*}
\]

6. (8pts) Match the equation and the description of the surface by writing the appropriate capital letter (A-D) in the provided blank. Each letter should be used exactly once.

(a) _____ In cylindrical coordinates, the surface \( z = r^2 \).
(b) _____ In cylindrical coordinates, the surface \( r^2 + z^2 = 4 \).
(c) _____ In spherical coordinates, the surface \( \rho = 2 \cos \phi \).
(d) _____ In spherical coordinates, the surface \( \theta = \frac{3\pi}{4} \).

A: a sphere centered at the origin.
B: a half-plane.
C: a paraboloid
D: a sphere centered at the point \((0, 0, 1)\) in Cartesian coordinates.

7. (9pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated. Please simplify as much as possible.

(a) Find the cylindrical coordinates of the point with Cartesian coordinates \((-1, 1, 3)\)

\[
\begin{align*}
\rho = & \ \_ & \_ \\
\theta = & \ \_ \_ & \_ \\
z = & \ \_ \_ \_ 
\end{align*}
\]

(b) Find the spherical coordinates of the point with Cartesian coordinates \((1, \sqrt{3}, -2)\)

\[
\begin{align*}
\rho = & \ \_ \_ & \_ \\
\theta = & \ \_ \_ \_ \\
\phi = & \ \_ \_ \_ 
\end{align*}
\]

(c) Find the Cartesian coordinates of the point with cylindrical coordinates \((2, \frac{\pi}{6}, \pi)\)

\[
\begin{align*}
x = & \ \_ \_ \_ \\
y = & \ \_ \_ \_ \\
z = & \ \_ \_ \_ 
\end{align*}
\]
8. (12pts) Evaluate the following limits. If they do not exist, write ‘DNE’ and explain why.

(a) \( \lim_{(x,y) \to (0,0)} \frac{e^{x^2+y^2}}{1 + x^2 + y^2} \)

(b) \( \lim_{(x,y) \to (0,0)} \frac{x^2 + y}{x^2 - y} \)

(c) \( \lim_{(x,y) \to (0,0)} \frac{x^3y}{x^2 + y^2} \) \textbf{Hint:} Use polar coordinates.

(d) \( \lim_{h \to 0} \frac{\sin((x+h)y) - \sin(xy)}{h} \) \textbf{Hint:} Think derivative.

9. (12pts) Consider the function \( f(x, y) = xy \cos(x^2) \).

Compute the following partial derivatives:

(a) \( f_x(x, y) = \)

(b) \( f_y(x, y) = \)

(c) \( f_{yy}(x, y) = \)

(d) Find \( \nabla f(0, 2) \). That is find the gradient of \( f \) at the point \( (0, 2) \).