2210-90 Exam 1
Spring 2014

Name _____

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (15pts) Consider the vectors $\mathbf{u} = \langle 1, -1, 5 \rangle$ and $\mathbf{v} = \langle 2, -2, 3 \rangle$. Find

   (a) (2pts) $5\mathbf{u}$

   (b) (2pts) $\mathbf{u} - \mathbf{v}$

   (c) (2pts) $||\mathbf{u}||$

   (d) (2pts) The unit vector which points in the same direction as $\mathbf{u}$

   (e) (2pts) $\mathbf{u} \cdot \mathbf{v}$

   (f) (2pts) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$. Answer in the form $\theta = \cos^{-1}(x)$.

   (g) (3pts) $\mathbf{u} \times \mathbf{v}$

2. (5pts) Find an equation of the plane containing the point $(1, -1, 3)$ which **never** intersects the plane

$$x - 2y + 5z = 9$$

3. (6pts) The line segment connecting $(0, 2, -4)$ to $(2, 2, 6)$ is a diameter of a sphere. Find an equation of this sphere.

4. (12pts) Suppose a particle's position at time t is given by the curve

$$\mathbf{r}(t) = 6t\mathbf{i} + 9\mathbf{j} + (t^2 + 1)\mathbf{k}.$$

(a) (2pts) Find the velocity $\mathbf{v}(t) = \mathbf{r}'(t)$ of the particle at time t.

(b) (3pts) Set up an integral which gives the arc length of the curve between times $t = 0$ and $t = 7$. **Do not evaluate**.

(c) (2pts) Find the acceleration $\mathbf{a}(t) = \mathbf{r}''(t)$ of the particle at time t.

(d) (5pts) Find the curvature $\kappa(t) = \frac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$ of the particle's path at time t.

5. (6pts) Suppose the velocity of a particle is given by

$$\mathbf{v}(t) = \langle \sin t, e^t, 3\cos t \rangle.$$

If the particle's initial position is $\mathbf{r}(0) = \langle 1, 1, 3 \rangle$, where is the particle at time $t = \pi$?

6. (14pts) Match the equation with the type of surface it determines by writing the appropriate capital letter (**A-G**) in the provided blank. Each letter should be used exactly once.

_____	$z = 9x - y + 7$	**A** Elliptic Paraboloid
_____	$y = x^2 + 5z^2$	**B** Ellipsoid
_____	$2x^2 + y^2 + 9z^2 = 3$	**C** Hyperboloid of one sheet
_____	$y^2 - x^2 - z = 1$	**D** Hyperboloid of two sheets
_____	$x^2 + y^2 + z^2 = 3$	**E** Hyperbolic Paraboloid
_____	$x^2 + 2y^2 - z^2 = 1$	**F** Plane
_____	$x^2 - 2y^2 - z^2 = 1$	**G** Sphere

7. (8pts) Match the equation and the description of the surface by writing the appropriate capital letter (**A-D**) in the provided blank. Each letter should be used exactly once.

(a) _____ In cylindrical coordinates, the surface $r = 4$.

(b) _____ In cylindrical coordinates, the surface $r^2 - z^2 = 4$.

(c) _____ In spherical coordinates, the surface $\phi = \frac{\pi}{2}$.

(d) _____ In spherical coordinates, the surface $\rho = 4$.

A a sphere.

B a plane.

C a cylinder.

D a hyperboloid.

8. (9pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated. Please simplify as much as possible.

(a) Find the cylindrical coordinates of the point with Cartesian coordinates $(-1, -1, 5)$

$r =$ _____ $\theta =$ _____ $z =$ _____

(b) Find the spherical coordinates of the point with Cartesian coordinates $(\sqrt{2}, \sqrt{2}, 2\sqrt{3})$

$\rho =$ _____ $\theta =$ _____ $\phi =$ _____

(c) Find the Cartesian coordinates of the point with cylindrical coordinates $(2, -\frac{\pi}{3}, \sqrt{3})$

$x =$ _____ $y =$ _____ $z =$ _____

9. (12pts) Evaluate the following limits. If they do not exist, write 'DNE' and explain why.

(a) $\lim_{(x,y)\to(0,0)} \ln(x^2+y^2+1)$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{(x+2y)^2}{x^2+4y^2}$

(c) $\lim_{(x,y)\to(0,0)} \dfrac{y^3}{x^2+y^2}$ **Hint:** Use polar coordinates.

10. (13pts) Consider the function

$$f(x,y) = xe^{-3y} + x^2y^2.$$

(a) (7pts) Find the equation of the tangent plane to the graph of $z = f(x,y)$ at the point $(1,0,1)$.

(b) (6pts) Find the following second derivatives:

i. $f_{xx}(x,y) =$ _____

ii. $f_{yy}(x,y) =$ _____

iii. $f_{xy}(x,y) =$ _____