Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (15pts) Consider the vectors $\mathbf{u} = \langle 1, -1, 5 \rangle$ and $\mathbf{v} = \langle 2, -2, 3 \rangle$. Find
   (a) (2pts) $5\mathbf{u}$
   (b) (2pts) $\mathbf{u} - \mathbf{v}$
   (c) (2pts) $||\mathbf{u}||$
   (d) (2pts) The unit vector which points in the same direction as $\mathbf{u}$
   (e) (2pts) $\mathbf{u} \cdot \mathbf{v}$
   (f) (2pts) Find the angle $\theta$ between $\mathbf{u}$ and $\mathbf{v}$. Answer in the form $\theta = \cos^{-1}(x)$.
   (g) (3pts) $\mathbf{u} \times \mathbf{v}$

2. (5pts) Find an equation of the plane containing the point $(1, -1, 3)$ which never intersects the plane $x - 2y + 5z = 9$.
3. (6pts) The line segment connecting (0, 2, −4) to (2, 2, 6) is a diameter of a sphere. Find an equation of this sphere.

4. (12pts) Suppose a particle’s position at time $t$ is given by the curve

$$r(t) = 6t\mathbf{i} + 9\mathbf{j} + (t^2 + 1)\mathbf{k}.$$  

(a) (2pts) Find the velocity $v(t) = r'(t)$ of the particle at time $t$.

(b) (3pts) Set up an integral which gives the arc length of the curve between times $t = 0$ and $t = 7$. Do not evaluate.

(c) (2pts) Find the acceleration $a(t) = r''(t)$ of the particle at time $t$.

(d) (5pts) Find the curvature $\kappa(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3}$ of the particle’s path at time $t$.

5. (6pts) Suppose the velocity of a particle is given by

$$v(t) = (\sin t, e^t, 3\cos t).$$

If the particle’s initial position is $r(0) = (1, 1, 3)$, where is the particle at time $t = \pi$?
6. (14pts) Match the equation with the type of surface it determines by writing the appropriate capital letter (A-G) in the provided blank. Each letter should be used exactly once.

\[
\begin{align*}
&\text{_____} \quad z = 9x - y + 7 \quad \text{A Elliptic Paraboloid} \\
&\text{_____} \quad y = x^2 + 5z^2 \quad \text{B Ellipsoid} \\
&\text{_____} \quad 2x^2 + y^2 + 9z^2 = 3 \quad \text{C Hyperboloid of one sheet} \\
&\text{_____} \quad y^2 - x^2 - z = 1 \quad \text{D Hyperboloid of two sheets} \\
&\text{_____} \quad x^2 + y^2 + z^2 = 3 \quad \text{E Hyperbolic Paraboloid} \\
&\text{_____} \quad x^2 + 2y^2 - z^2 = 1 \quad \text{F Plane} \\
&\text{_____} \quad x^2 - 2y^2 - z^2 = 1 \quad \text{G Sphere}
\end{align*}
\]

7. (8pts) Match the equation and the description of the surface by writing the appropriate capital letter (A-D) in the provided blank. Each letter should be used exactly once.

(a) _____ In cylindrical coordinates, the surface \( r = 4 \).
(b) _____ In cylindrical coordinates, the surface \( r^2 - z^2 = 4 \).
(c) _____ In spherical coordinates, the surface \( \phi = \frac{\pi}{2} \).
(d) _____ In spherical coordinates, the surface \( \rho = 4 \).

A a sphere.
B a plane.
C a cylinder.
D a hyperboloid.

8. (9pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated. Please simplify as much as possible.

(a) Find the cylindrical coordinates of the point with Cartesian coordinates \((-1, -1, 5)\)

\[
\begin{align*}
r &= \underline{\quad} \\
\theta &= \underline{\quad} \\
z &= \underline{\quad}
\end{align*}
\]

(b) Find the spherical coordinates of the point with Cartesian coordinates \((\sqrt{2}, \sqrt{2}, 2\sqrt{3})\)

\[
\begin{align*}
\rho &= \underline{\quad} \\
\theta &= \underline{\quad} \\
\phi &= \underline{\quad}
\end{align*}
\]

(c) Find the Cartesian coordinates of the point with cylindrical coordinates \((2, -\frac{\pi}{4}, \sqrt{3})\)

\[
\begin{align*}
x &= \underline{\quad} \\
y &= \underline{\quad} \\
z &= \underline{\quad}
\end{align*}
\]
9. (12pts) Evaluate the following limits. If they do not exist, write ‘DNE’ and explain why.

(a) \[ \lim_{{(x,y) \to (0,0)}} \ln(x^2 + y^2 + 1) \]

(b) \[ \lim_{{(x,y) \to (0,0)}} \frac{(x+2y)^2}{x^2 + 4y^2} \]

(c) \[ \lim_{{(x,y) \to (0,0)}} \frac{y^3}{x^2 + y^2} \] \textbf{Hint:} Use polar coordinates.

10. (13pts) Consider the function \( f(x,y) = xe^{-3y} + x^2y^2 \).

(a) (7pts) Find the equation of the tangent plane to the graph of \( z = f(x,y) \) at the point \((1,0,1)\).

(b) (6pts) Find the following second derivatives:

i. \( f_{xx}(x,y) = \) __________________________

ii. \( f_{yy}(x,y) = \) __________________________

iii. \( f_{xy}(x,y) = \) __________________________