2210-90 Exam 1

Spring 2013

Name ____

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (16pts) Consider the vectors $\mathbf{u} = \langle 0, 2, -1 \rangle$ and $\mathbf{v} = \langle 3, 2, 3 \rangle$. Find

   (a) (2pts) $\mathbf{u} - 3\mathbf{v}$

   (b) (2pts) $||\mathbf{u}||$

   (c) (2pts) $\mathbf{u} \cdot \mathbf{v}$

   (d) (2pts) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$. Answer in the form $\theta = \cos^{-1}(x)$.

   (e) (4pts) $\mathbf{u} \times \mathbf{v}$

   (f) (4pts) Find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$

2. (8pts) Find a unit vector which is perpendicular to both of the vectors $\mathbf{a} = 2\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i}-2\mathbf{j}+\mathbf{k}$.

3. (12pts) Consider the points $P(1, 3, -1)$ and $Q(2, 2, 5)$.

   (a) (3pts) Find the vector $\overrightarrow{PQ}$ that points in the direction from P to Q; that is find the vector with a tail at P and head at Q.

   (b) (3pts) Find the midpoint of P and Q.

   (c) (6pts) Find the equation of the plane which is equidistant from P and Q. **Hint:** This plane has normal vector $\overrightarrow{PQ}$ and contains the midpoint.

4. (8pts) The set of all points (x, y, z) which satisfy

$$x^2 + y^2 + z^2 - 4x + 6y - 16z = 0$$

determines a sphere. Determine the center point (h, k, l) and the radius r of the sphere. Write your final answers in the provided blanks. **Hint:** Complete the squares.

$h =$____________

$k =$____________

$l =$____________

$r =$____________

5. (20pts) Suppose a particle's position at time t is given by the curve

$$\mathbf{r}(t) = 2\cos(2t)\mathbf{i} + 2\sin(2t)\mathbf{j} + 3t\mathbf{k}.$$

(a) (2pts) Find the velocity $\mathbf{v}(t)$ of the particle at time t.

(b) (3pts) Find the arc length of the curve between times $t = 0$ and $t = 3$.

(c) (2pts) Find the acceleration $\mathbf{a}(t)$ of the particle at time t.

(d) (2pts) Find the unit tangent vector $\mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||}$.

(e) (3pts) Find the principal unit normal vector $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}$.

(f) (6pts) Find the curvature $\kappa(t)$ of the particle's path at time t.

(g) (2pts) For all time t, this curve is contained on what type of surface? Circle the correct letter

A a sphere of radius 3 centered at the origin

B a hyperboloid of one sheet parallel to the y-axis

C a cylinder of radius 2 parallel to the z-axis

6. (16pts) Match the equation with its graph on the right by writing the appropriate capital letter (**A-H**) in the provided blank.

(a) ______ $2x^2 + 2y^2 + z^2 = 1$

(b) ______ $x^2 + z^2 - y^2 = -1$

(c) ______ $y^2 = x^2 + z^2$

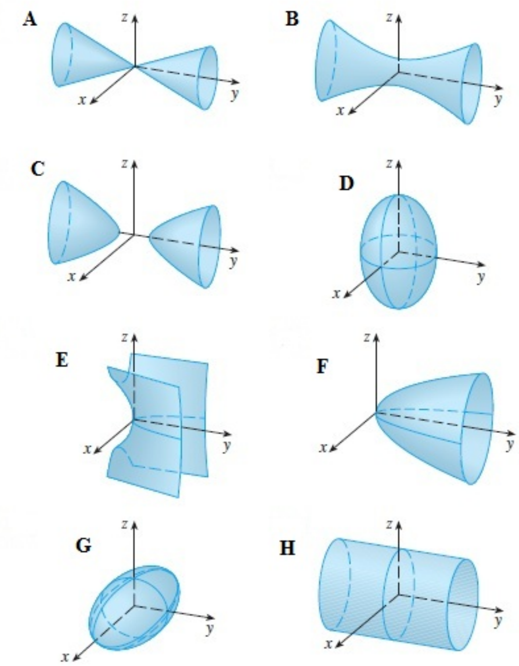
(d) ______ $x^2 + z^2 = 1$

(e) ______ $y = x^2 + z^2$

(f) ______ $x^2 + 2y^2 + 2z^2 = 1$

(g) ______ $x^2 + z^2 - y^2 = 1$

(h) ______ $y = x^2 - z^2$



7. (8pts) Match the equation and the description of the surface by writing the appropriate capital letter (**A-D**) in the provided blank. Each letter should be used exactly once.

(a) ______ In cylindrical coordinates, the surface $r = 5$.

(b) ______ In cylindrical coordinates, the surface $z = r$.

(c) ______ In spherical coordinates, the surface $\rho = 1$.

(d) ______ In spherical coordinates, the surface $\phi = \frac{3\pi}{4}$.

A sphere centered at the origin.

B cylinder parallel to the z-axis.

C cone opening in the positive z direction.

D cone opening in the negative z direction.

8. (12pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated.

(a) Find the cylindrical coordinates of the point with Cartesian coordinates $(\sqrt{3}, 1, -2)$

$r =$ ____________ $\theta =$ ____________ $z =$ ____________

(b) Find the spherical coordinates of the point with Cartesian coordinates $(1, 1, \sqrt{2})$

$\rho =$ ____________ $\theta =$ ____________ $\phi =$ ____________

(c) Find the Cartesian coordinates of the point with spherical coordinates $(2, \frac{\pi}{6}, \frac{\pi}{2})$

$x =$ ____________ $y =$ ____________ $z =$ ____________

(d) Find the Cartesian coordinates of the point with cylindrical coordinates $(4\sqrt{2}, -\frac{\pi}{4}, \pi)$

$x =$ ____________ $y =$ ____________ $z =$ ____________