Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Page 5 is blank in case you need extra paper. Please circle your final answers.

1. (16pts) Consider the vectors $u = \langle 0, 2, -1 \rangle$ and $v = \langle 3, 2, 3 \rangle$. Find
   
   (a) (2pts) $u - 3v$

   (b) (2pts) $||u||$

   (c) (2pts) $u \cdot v$

   (d) (2pts) Find the angle $\theta$ between $u$ and $v$. Answer in the form $\theta = \cos^{-1}(x)$.

   (e) (4pts) $u \times v$

   (f) (4pts) Find the vector projection of $u$ onto $v$

2. (8pts) Find a unit vector which is perpendicular to both of the vectors $a = 2i + j - k$ and $b = 5i - 2j + k$. 


3. (12pts) Consider the points $P(1, 3, -1)$ and $Q(2, 2, 5)$.

(a) (3pts) Find the vector $\overrightarrow{PQ}$ that points in the direction from $P$ to $Q$; that is find the vector with a tail at $P$ and head at $Q$.

(b) (3pts) Find the midpoint of $P$ and $Q$.

(c) (6pts) Find the equation of the plane which is equidistant from $P$ and $Q$. **Hint:** This plane has normal vector $\overrightarrow{PQ}$ and contains the midpoint.

4. (8pts) The set of all points $(x, y, z)$ which satisfy

$$x^2 + y^2 + z^2 - 4x + 6y - 16z = 0$$

determines a sphere. Determine the center point $(h, k, l)$ and the radius $r$ of the sphere. Write your final answers in the provided blanks. **Hint:** Complete the squares.

$h = \underline{\hspace{2cm}}$

$k = \underline{\hspace{2cm}}$

$l = \underline{\hspace{2cm}}$

$r = \underline{\hspace{2cm}}$
5. (20pts) Suppose a particle’s position at time $t$ is given by the curve

$$\mathbf{r}(t) = 2 \cos (2t) \mathbf{i} + 2 \sin (2t) \mathbf{j} + 3t \mathbf{k}.$$ 

(a) (2pts) Find the velocity $\mathbf{v}(t)$ of the particle at time $t$.

(b) (3pts) Find the arc length of the curve between times $t = 0$ and $t = 3$.

(c) (2pts) Find the acceleration $\mathbf{a}(t)$ of the particle at time $t$.

(d) (2pts) Find the unit tangent vector $\mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||}$.

(e) (3pts) Find the principal unit normal vector $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}$.

(f) (6pts) Find the curvature $\kappa(t)$ of the particle’s path at time $t$.

(g) (2pts) For all time $t$, this curve is contained on what type of surface? Circle the correct letter

A  a sphere of radius 3 centered at the origin
B  a hyperboloid of one sheet parallel to the $y$-axis
C  a cylinder of radius 2 parallel to the $z$-axis
6. (16pts) Match the equation with its graph on the right by writing the appropriate capital letter (A-H) in the provided blank.

(a) ______ $2x^2 + 2y^2 + z^2 = 1$
(b) ______ $x^2 + z^2 - y^2 = -1$
(c) ______ $y^2 = x^2 + z^2$
(d) ______ $x^2 + z^2 = 1$
(e) ______ $y = x^2 + z^2$
(f) ______ $x^2 + 2y^2 + 2z^2 = 1$
(g) ______ $x^2 + z^2 - y^2 = 1$
(h) ______ $y = x^2 - z^2$

7. (8pts) Match the equation and the description of the surface by writing the appropriate capital letter (A-D) in the provided blank. Each letter should be used exactly once.

(a) ______ In cylindrical coordinates, the surface $r = 5$.
(b) ______ In cylindrical coordinates, the surface $z = r$.
(c) ______ In spherical coordinates, the surface $\rho = 1$.
(d) ______ In spherical coordinates, the surface $\phi = \frac{3\pi}{4}$.

A sphere centered at the origin.
B cylinder parallel to the z-axis.
C cone opening in the positive z direction.
D cone opening in the negative z direction.

8. (12pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated.

(a) Find the cylindrical coordinates of the point with Cartesian coordinates $(\sqrt{3}, 1, -2)$
   \[ r = \quad \theta = \quad z = \]

(b) Find the spherical coordinates of the point with Cartesian coordinates $(1, 1, \sqrt{2})$
   \[ \rho = \quad \theta = \quad \phi = \]

(c) Find the Cartesian coordinates of the point with spherical coordinates $(2, \frac{\pi}{6}, \frac{\pi}{3})$
   \[ x = \quad y = \quad z = \]

(d) Find the Cartesian coordinates of the point with cylindrical coordinates $(4\sqrt{2}, -\frac{\pi}{4}, \pi)$
   \[ x = \quad y = \quad z = \]