2210-90 Exam 1

Fall 2013

Name ____

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Please circle your final answers.

1. (10pts) Consider the vectors $\mathbf{u} = \langle 1, 5, -1\rangle$ and $\mathbf{v} = \langle 4, 1, 1\rangle$. Find

   (a) (2pts) $\mathbf{v} + 3\mathbf{u}$

   (b) (2pts) $||\mathbf{u}||$

   (c) (3pts) $\mathbf{u} \cdot \mathbf{v}$

   (d) (3pts) $\mathbf{u} \times \mathbf{v}$

2. (7pts) Find a unit vector that is orthogonal to the plane containing the points $P = (1, 1, -1)$, $Q = (2, 0, 1)$, and $R = (0, 0, 5)$.

3. (6pts) The following equation determines a sphere. Find the center and radius of the sphere. **Hint:** Complete the squares.

$$x^2 + y^2 + z^2 + 2x - 6y = -6$$

   Center= (______ , ______, ______)

   Radius= ______

4. (6pts) Find a vector equation for a line that passes through the point $(1, 2, 3)$ at $t = 0$ and the point $(-1, 4, -2)$ at $t = 1$. Note, your answer should be a curve

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$$

5. (16pts) Suppose a particle's position at time t is given by the curve

$$\mathbf{r}(t) = 5t\mathbf{i} + \sin(3t)\mathbf{j} - \cos(3t)\mathbf{k}.$$

For this problem, it may be helpful if you remember the trig identity $\sin^2 x + \cos^2 x = 1$.

(a) (3pts) Find the velocity $\mathbf{v}(t)$ of the particle at time t.

(b) (3pts) Find the acceleration $\mathbf{a}(t)$ of the particle at time t.

(c) (3pts) Find the arc length of the curve between times $t = 0$ and $t = 3$.

(d) (3pts) Find the unit tangent vector $\mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||}$.

(e) (4pts) Find the curvature $\kappa(t) = \frac{||\mathbf{T}'(t)||}{||\mathbf{v}(t)||}$ of the particle's path at time t.

6. (21pts, 1.5pts each) Match the equation with the type of surface it determines by writing the appropriate capital letter (**A-I**) in the provided blank. The last six equations are written in cylindrical or spherical coordinates. Some letters may be used more than once.

______	$x^2 + y^2 - z^2 = 1$	**A** Elliptic Paraboloid
______	$x^2 + z^2 - y = 1$	**B** Ellipsoid
______	$x^2 + y^2 + \frac{z^2}{3} = 1$	**C** Hyperboloid of one sheet
______	$x^2 - 2y^2 - z^2 = 1$	**D** Hyperboloid of two sheets
______	$z = x^2 - y^2$	**E** Hyperbolic Paraboloid
______	$x^2 + y^2 + z^2 = 1$	**F** Plane
______	$x = 2y - z$	**G** Sphere
______	$x^2 + y^2 = 1$	**H** Cylinder
______	$r^2 = z^2$	**I** Cone
______	$r = 1$	
______	$2r^2 + z^2 = 1$	
______	$\phi = \frac{\pi}{5}$	
______	$\rho = 2$	
______	$\rho \cos \phi = 1$	

7. (9pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated. Please simplify as much as possible.

   (a) Find the spherical coordinates of the point with Cartesian coordinates $(-1, 1, \sqrt{2})$

   $\rho =$ ____________ $\theta =$ ____________ $\phi =$ ____________

   (b) Find the cylindrical coordinates of the point with Cartesian coordinates $(1, \sqrt{3}, -2)$

   $r =$ ____________ $\theta =$ ____________ $z =$ ____________

   (c) Find the Cartesian coordinates of the point with spherical coordinates $(4, \frac{\pi}{3}, \frac{2\pi}{3})$

   $x =$ ____________ $y =$ ____________ $z =$ ____________

8. (12pts) Evaluate the following limits. If they do not exist, write 'DNE' and explain why.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\cos(x^2)}{\sqrt{1-x^2-y^2}}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+2y}{x-y}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{-y}{x^2+y^2}$ **Hint:** Use polar coordinates.

9. (13pts) Consider the function

$$f(x,y) = x^2 \sin(xy) - x.$$

(a) (7pts) Find the equation of the tangent plane to the graph of $z = f(x,y)$ at the point $(1,0,-1)$.

(b) (6pts) Find the following second derivatives:

i. $f_{xx}(x,y) =$ ______________________________

ii. $f_{yy}(x,y) =$ ______________________________

iii. $f_{xy}(x,y) =$ ______________________________