2210-90 Exam 1
Fall 2012

Name _____

Instructions. Show all work and include appropriate explanations when necessary. Please try to do all all work in the space provided. Page 5 is blank in case you need extra paper. Please circle your final answer.

1. (21pts) Consider the vectors $\mathbf{u} = \langle 1, 2, -3 \rangle$ and $\mathbf{v} = \langle 1, 0, 2 \rangle$. Find

   (a) (2pts) $\mathbf{u} - 5\mathbf{v}$

   (b) (2pts) $||\mathbf{u}||$

   (c) (2pts) The unit vector which points in the same direction as $\mathbf{u}$

   (d) (2pts) $\mathbf{u} \cdot \mathbf{v}$

   (e) (1pt) Are $\mathbf{u}$ and $\mathbf{v}$ orthogonal? Circle one: YES NO

   (f) (3pts) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

   (g) (3pts) $\mathbf{u} \times \mathbf{v}$

   (h) (2pts) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{u})$

   (i) (4pts) Find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$

2. (10pts) Find an equation for the plane which contains the points $(1, 1, 0)$, $(2, 2, -1)$, and $(-3, 5, 2)$.

3. (8pts) Find the equation for the sphere which is centered at $(-3, 4, 1)$ and contains the point (1,0,-1).

4. (8pts) Find a parametric equation for a line $\mathbf{r}(t)$ that passes through the point $(2, 1, 3)$ at $t = 0$ and passes through the point $(1, -4, 6)$ at $t = 1$.

5. (8pts) Find the arclength of the curve

$$\mathbf{r}(t) = t\mathbf{i} + \frac{1}{3}t^3\mathbf{j} + \frac{\sqrt{2}}{2}t^2\mathbf{k}.$$

for $0 \leq t \leq 2$.

6. (12pts) Suppose a particle's position at time t is given by the curve

$$\mathbf{r}(t) = t\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}.$$

(a) (2pts) Find the velocity $\mathbf{v}(t)$ of the particle at time t.

(b) (2pts) Find the acceleration $\mathbf{a}(t)$ of the particle at time t.

(c) (6pts) Find the curvature $\kappa(t)$ of the particle's path at time t.

(d) (2pts) For all time t, this curve is contained on what type of surface? Circle the correct letter

A a sphere of radius 1 centered at the origin

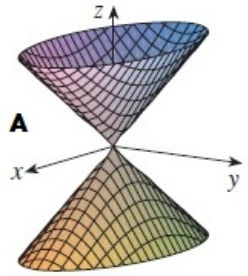
B a cylinder of radius 1 parallel to the z-axis

C a paraboloid opening in the positive z direction

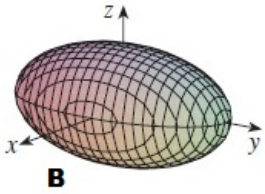
7. (18 pts) Match the equation with the type of surface it describes by writing the appropriate capital letter (**A-F**) in the provided blank (**Note:** The last three equations are written using cylindrical coordinates).

(a) _____ $z = y^2 - x^2$
(b) _____ $z^2 = x^2 + y^2$
(c) _____ $4x^2 + y^2 + 4z^2 = 1$
(d) _____ $-x^2 - 5y^2 + z^2 = 1$
(e) _____ $z = 5x^2 + y^2$
(f) _____ $3x^2 + y^2 - 5z^2 = 3$
(g) _____ $z = r^2$
(h) _____ $|z| = r$
(i) _____ $r^2 - z^2 = 1$

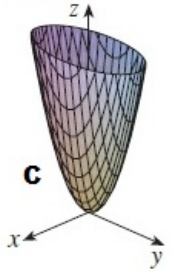
Cone



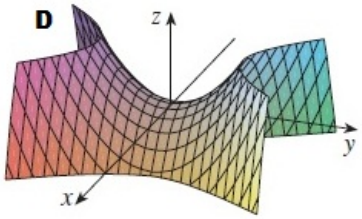
Ellipsoid



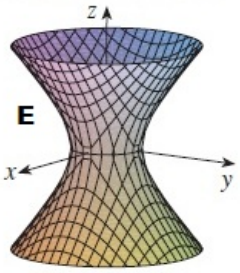
Elliptic Paraboloid



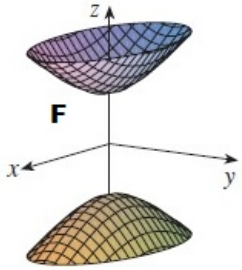
Hyperbolic Paraboloid



Hyperboloid of One Sheet



Hyperboloid of Two Sheets



8. (15pts) Convert between Cartesian, cylindrical, and spherical coordinates as indicated

(a) Find the cylindrical coordinates of the point with Cartesian coordinates $(2, -2, 3)$

$r =$ _____ $\theta =$ _____ $z =$ _____

(b) Find the spherical coordinates of the point with Cartesian coordinates $(0, 5, 0)$

$\rho =$ _____ $\theta =$ _____ $\phi =$ _____

(c) Find the Cartesian coordinates of the point with spherical coordinates $(3, \frac{\pi}{6}, \frac{3\pi}{4})$

$x =$ _____ $y =$ _____ $z =$ _____

(d) Find the Cartesian coordinates of the point with cylindrical coordinates $(2, -\frac{\pi}{3}, -5)$

$x =$ _____ $y =$ _____ $z =$ _____

(e) Find the spherical coordinates of the point with cylindrical coordinates $(1, -\frac{\pi}{4}, 0)$

$\rho =$ _____ $\theta =$ _____ $\phi =$ _____