2210-90 Final Exam

Summer 2013

Name ____

Instructions. Show all work and include appropriate explanations when necessary. Correct answers unaccompanied by work may not receive full credit. Please try to do all all work in the space provided and circle your final answer.

1. (15pts) Consider the three vectors $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$

   (a) (3pts) Find $\mathbf{u} + \mathbf{v} - \mathbf{w}$

   (b) (3pts) Find $||\mathbf{v}||$

   (c) (3pts) Find $\mathbf{w} \cdot \mathbf{u}$

   (d) (2pts) Which two vectors are orthogonal? Circle one:

   A. $\mathbf{u}, \mathbf{v}$　　　B. $\mathbf{u}, \mathbf{w}$　　　C. $\mathbf{v}, \mathbf{w}$

   (e) (4pts) Find $\mathbf{u} \times \mathbf{w}$

2. (9pts) Find an equation for the plane which contains the points $(1, 0, 1)$, $(2, -1, 1)$, and $(3, 1, 2)$.

3. (20pts) Consider the function $f(x, y) = \frac{1}{2}x^2 - xy + \frac{1}{3}y^3$.

   (a) (4pts) Find $\nabla f(x, y)$, the gradient of f.

   (b) (4pts) Find $(D_{\mathbf{u}}f)(2, 1)$, the directional derivative of f at $(2, 1)$ in the direction $\mathbf{u} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$.

   (c) (4pts) Find the equation of the tangent plane to the graph of f at the point $(2, 1)$.

   (d) (4pts) Find the two critical points of f.

   (e) (4pts) Is the critical point with the larger x-coordinate a local minimum, a local maximum, or a saddle point?

4. (10pts) Use the method of Lagrange multipliers to find the point on the hyperbola $x^2 - y^2 = 1$ in the first quadrant which is closest to the point $(0, 1)$. Do this by finding the point (x, y) in the first quadrant which minimizes the function $f(x, y) = x^2 + (y - 1)^2$, the distance squared from the point $(0, 1)$, subject to the constraint $g(x, y) = x^2 - y^2 - 1 = 0$.

5. (24pts) Evaluate the following double or triple integrals:

(a) (6pts) $\iint_R (2xy + 1)\, dA$, where R is the rectangle $0 \le x \le 2$, $0 \le y \le 3$.

(b) (6pts) $\iint_T (x^2 + 3y)\, dA$, where T is the triangle in the xy-plane with vertices $(0,0)$, $(2,0)$ and $(2,2)$.

(c) (6pts) $\iiint_B (2x - 4y + z^2)\, dV$, where B is the box $0 \le x \le 1$, $-1 \le y \le 1$, and $0 \le z \le 3$.

(d) (6pts) $\iiint_H (z^2(x^2 + y^2 + z^2))\, dV$, where H is the hemisphere $x^2 + y^2 + z^2 \le 1$, $y \ge 0$.

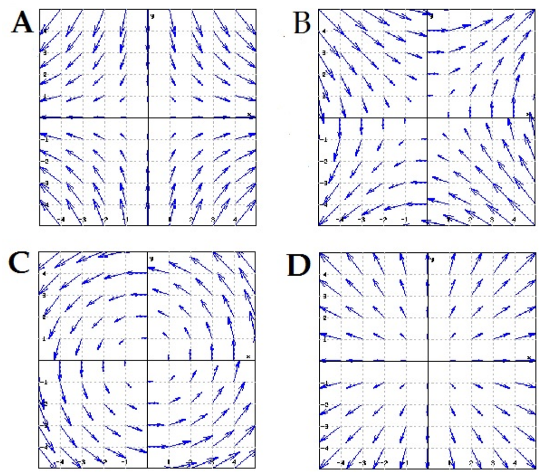
6. (8pts) Match the following vector fields with the graphs below by writing the letter in the blank provided.

______ $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$

______ $\mathbf{F}(x, y) = x\mathbf{i} - y\mathbf{j}$

______ $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$

______ $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$



7. (10pts) Consider the vector field $\mathbf{F}(x, y) = xy^2\mathbf{i} - 2xy\mathbf{j}$.

(a) (4pts) Compute $\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$, where C_1 is the curve connecting $(0, 0)$ to $(1, 1)$ parameterized by

$$x(t) = t \qquad y(t) = t$$

for $0 \leq t \leq 1$.

(b) (4pts) Compute $\displaystyle\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$, where C_2 is the curve connecting $(0, 0)$ to $(1, 1)$ parameterized by

$$x(t) = t^2 \qquad y(t) = t$$

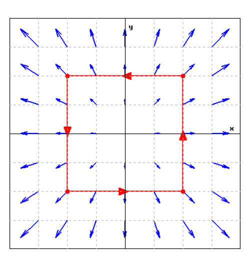
for $0 \leq t \leq 1$.

(c) (2pts) Is $\mathbf{F}$ a conservative vector field? Circle one: YES NO

8. (6pts) Consider the vector field and the closed oriented curve C drawn below. In the blank provided, write 'P' if the quantity is positive, 'N' if the quantity is negative, and 'Z' if the quantity is zero.

______ $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$

______ $\displaystyle\int_C \mathbf{F} \cdot \mathbf{n}\, ds$



9. (16pts) Let $\mathbf{F}(x, y) = (x^3 - y)\mathbf{i} + (y^3 + x)\mathbf{j}$ and let C denote the unit circle traversed counterclockwise.

   (a) (8pts) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Green's Theorem.

   (b) (8pts) Compute the line integral $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$ using the plane Divergence Theorem (which is also a form of Green's Theorem).

10. (8pts) Consider the vector field $\mathbf{F}(x, y, z) = \frac{1}{3}zx^3\mathbf{i} + \frac{1}{3}zy^3\mathbf{j} + xy\mathbf{k}$. Use the Divergence Theorem and to compute

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS,$$

where S is the surface of the cylinder $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$ and $\mathbf{n}$ is the outward pointing unit normal vector.

11. (8pts) Let $\mathbf{F}(x, y, z) = (x^2y + 1)\mathbf{i} + (3y^2 + xz)\mathbf{j} - xz^2\mathbf{k}$.

(a) (4pts) Compute $\operatorname{div} \mathbf{F}$

(b) (4pts) Compute $\operatorname{curl} \mathbf{F}$

12. (16pts) Let S denote the top half of the unit sphere, a surface determined by the graph of

$$z = \sqrt{1 - x^2 - y^2}.$$

(a) (8pts) Find the surface area of S by evaluating an integral.

(b) (8pts) Use Stokes's Theorem to evaluate

$$\iint_S (\operatorname{curl} \mathbf{F}) \cdot \mathbf{n} \, dS,$$

where $\mathbf{F}$ denotes the vector field

$$\mathbf{F} = (x^2 + z)\mathbf{i} + yz\mathbf{j} + xy\mathbf{k}.$$