2210-90 Final Exam
Fall 2012

Name ____

Instructions. Show all work and include appropriate explanations when necessary. Please try to do all all work in the space provided. Please circle your final answer.

1. (15pts) Let $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

   (a) (3pts) Find $\mathbf{u} - \mathbf{v}$

   (b) (4pts) Find $||\mathbf{u}||$

   (c) (4pts) Find $\mathbf{u} \cdot \mathbf{v}$

   (d) (4pts) Find $\mathbf{u} \times \mathbf{v}$

2. (10pts) Find the equation for the plane which contains the points $(1, 0, 1)$, $(3, -1, -1)$, and $(2, 5, 0)$.

3. (15pts) Consider the function $f(x, y) = x\sqrt{y}$.

   (a) (5pts) Find ∇f, the gradient of f.

   (b) (5pts) Find the equation of the tangent plane to the graph of f at the point $(3, 4)$.

   (c) (5pts) Use part (b) above to estimate the value of $(3 + \frac{1}{3})\sqrt{4 + \frac{2}{3}}$.

4. (12pts) Use Lagrange multipliers to find the extreme values (both maximum and minimum) of the function $f(x, y) = xy$ on the ellipse $x^2 + 4y^2 = 1$. **Hint:** Solve for λ, then use the constraint equation.

5. Evaluate the following double integrals:

(a) (5pts) $\iint_R (x^2 + 2y)\, dA$, where R is the rectangle $0 \leq x \leq 1, 2 \leq y \leq 3$.

(b) (5pts) $\iint_R x\, dA$, where R is the region bounded by the x-axis, the y-axis, and the line $y = 1 - x$.

(c) (5pts) $\iint_R \sin(x^2 + y^2)\, dA$, where R is the region $x^2 + y^2 \leq \pi$.

6. Evaluate the following triple integrals:

(a) (5pts) $\iiint_R (x^3 z + y)\, dV$, where R is the rectangle $0 \le x \le 1, 0 \le y \le 1$, and $1 \le z \le 3$.

(b) (5pts) $\iiint_R \frac{z}{\sqrt{x^2+y^2}}\, dV$, where R is the cylinder $x^2 + y^2 \le 1$, $0 \le z \le 2$.

(c) (5pts) $\iiint_R z\, dV$, where R is the hemisphere $x^2 + y^2 + z^2 \le 1$, $z \ge 0$.

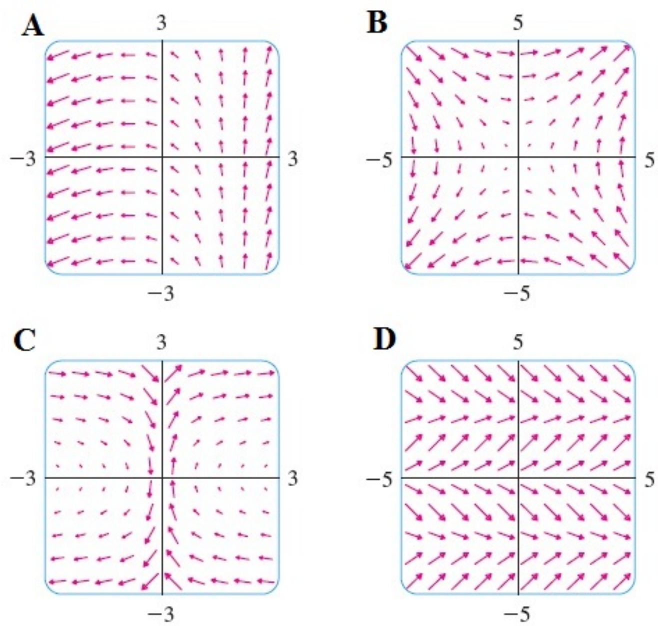
7. (8pts) Match the equation of the vector field with its plot by writing the letter in the blank provided.

______ $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$

______ $\mathbf{F}(x, y) = \mathbf{i} + \sin y\mathbf{j}$

______ $\mathbf{F}(x, y) = (x - 2)\mathbf{i} + (x + 1)\mathbf{j}$

______ $\mathbf{F}(x, y) = y\mathbf{i} + \frac{1}{x}\mathbf{j}$



8. (12pts) Consider the vector field $\mathbf{F}(x, y) = (2x - 2y)\mathbf{i} + (4y - 2x)\mathbf{j}$.

(a) (6pts) Find a function $f(x, y)$ such that $\nabla f = \mathbf{F}$.

(b) (6pts) Let C denote the curve with the parameterization

$$\mathbf{r}(t) = (1 + 2t)\mathbf{i} + (e^t \sin(2\pi t) - t^2)\mathbf{j}$$

for $0 \leq t \leq 1$. Use part (a) above to find

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

9. (10pts) Consider the vector field $\mathbf{F}(x, y) = xz\mathbf{i} + xyz\mathbf{j} - y^2\mathbf{k}$.

   (a) (4pts) Find $\operatorname{div} \mathbf{F}$

   (b) (4pts) Find $\operatorname{curl} \mathbf{F}$

   (c) (2pts) Is $\mathbf{F}$ a conservative vector field? Circle one: YES NO

10. (18pts) Let $\mathbf{F}(x, y) = (-2y + x)\mathbf{i} + (2x)\mathbf{j}$, and let C denote the counter-clockwise unit circle with parameterization

$$\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$$

for $0 \leq t \leq 2\pi$.

   (a) (6pts) Compute directly the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

   (b) (6pts) Use Green's Theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

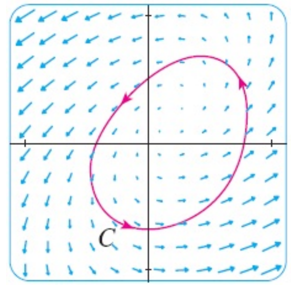
   (c) (6pts) Use the Divergence Theorem (plane version) to compute $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$.

11. (6pts) Indicate whether each statement is true (T) or false (F) by writing in the blank provided.

______ $\int_C \mathbf{F} \cdot d\mathbf{r} > 0$

______ $\int_C \mathbf{F} \cdot \mathbf{n}\, ds < 0$

______ $\mathbf{F}$ is conservative.



12. (14pts) Let S denote the surface determined by

$$z = 1 - x^2 - y^2,$$

where $x^2 + y^2 \leq 1$ (S is the graph of $f(x, y) = 1 - x^2 - y^2$ over the unit circle).

(a) (7pts) Find the surface area of S.

(b) (7pts) Use Stokes Theorem to evaluate

$$\iint_S (\operatorname{curl} \mathbf{F}) \cdot \mathbf{n}\, dS,$$

where $\mathbf{F}$ denotes the vector field

$$\mathbf{F} = y\mathbf{i} - x\mathbf{j} + xy\mathbf{k}.$$